

## MATHEMATICAL DESCRIPTION OF THE WOOD THERMAL CONDUCTIVITY ABOVE THE HYGROSCOPIC RANGE DURING WOOD FREEZING

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### ABSTRACT

A mathematical description of the wood thermal conductivity  $\lambda$  of wood above the hygroscopic range during freezing of wood has been suggested. The description takes into account the physics of the freezing process of both the free and the hygroscopically bound water in wood. It reflects also the influence of the fiber saturation point of wood species on the value of their  $\lambda$  during wood freezing and the influence of the temperature on the fiber saturation point of the wood during its freezing.

A software program has been prepared for the computation of  $\lambda$  according to the suggested mathematical description, which has been input in the calculation environment of Visual Fortran Professional. With the help of the program, computations have been done for the determination of the thermal conductivity in radial direction of often used in veneer and plywood production beech and poplar wood with moisture content  $0.4 \text{ kg}\cdot\text{kg}^{-1} \leq u \leq 1.2 \text{ kg}\cdot\text{kg}^{-1}$  at the temperature range between  $0 \text{ }^\circ\text{C}$  and  $-60 \text{ }^\circ\text{C}$  during freezing of the wood.

The obtained results can be used for modeling of the wood freezing process and for technological analysis of processes of thermal and hydrothermal treatment of frozen wood materials, as well as in software of systems for model based automatic control of such treatment.

**Key words:** wood, freezing, thermal conductivity, mathematical description, bound water, free water, computation.

### INTRODUCTION

It is known, that the wood thermal conductivity  $\lambda$  characterizes the intensity of the heat distribution in the wood materials. Because of this, for the modeling and calculation of the freezing processes in wood materials at given initial and boundary conditions, the knowledge of the wood thermal conductivity of the wood during its freezing is needed.

From the view point of the theory of heat conduction, the moist wood represents a 3-component dispersion capillary porous material, which includes a wood substance, water, and air. The thermal conductivity of each of the three components, as well as of the water and the ice, is different. For example, the thermal conductivity of the water at temperature  $t = 0 \text{ }^\circ\text{C}$  is equal to  $0.551 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ , and of the ice at the same temperature it is  $1.047 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ , i.e. almost twice as large and it increases to  $2.780 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $t = -50 \text{ }^\circ\text{C}$  (CHUDINOV 1966, 1968).

That is why the wood moisture content  $u$  is one of the most important factors, which influences the wood thermal conductivity. The replacement of the air in the wood pores by water causes a significant increase in  $\lambda$  according to a complex dependency. The wood thermal conductivity increases if the water in the wood becomes ice.

According to the theory of wood thermal treatment, the impact of the temperature on the thermal conductivity of the wood, which contains ice, is different in comparison to the moist wood without ice. If  $t$  decreases, the thermal conductivity of wood with ice, increases (KANTER 1955, CHUDINOV 1966, 1988, SHTEINHAGEN 1977, 1986, 1991, SHUBIN 1990, TREBULA – KLEMENT 2002). A reason for this on one hand is the decrease with decreasing of  $t$  of the quantity of non-frozen hygroscopically bound water in the wood, which has a significantly smaller  $\lambda$  according to that of the ice, and on another hand – the increase of  $\lambda$  of the ice in the wood when  $t$  decreases. The increase of  $t$  in the wood, which does not contain ice, causes an increase of its  $\lambda$  (Maclean 1941, KOLLMANN 1951, KANTER 1955, VORREITER 1958, STAMM 1964, CHUDINOV 1966, TENWOLDE *et al.* 1981, Požgaj *et al.* 1997, VIDELOV 2003, KUDELA 2005).

The wood density, which indirectly reflects its porosity, also has a significant impact on  $\lambda$ . The change of the wood density causes the change of the partial participation of the separate components of the wood, for which the thermal conductivity is different. With the increase of the density, i.e. with the decrease of the porosity,  $\lambda$  increases (VORREITER 1951, CHUDINOV 1966, 1968, DELIISKI 1977, 2003, 2011, YU *et al.* 2011). Not only the porosity, but also the form, dimensions, and position of the pores influence  $\lambda$ . Since the dimensions of the pores are different for the separate anatomical directions of the wood, this causes anisotropy of the wood thermal conductivity (CHUDINOV 1968, GU 2001, NIEMZ *et al.* 2010).

Besides this the precise determination of the wood thermal conductivity needs to take into account the impact of the fiber saturation point of the wood,  $u_{fsp}$ , which for the various wood species changes in a large range between  $0.2 \text{ kg}\cdot\text{kg}^{-1}$  and  $0.4 \text{ kg}\cdot\text{kg}^{-1}$  (KOLLMANN 1951, STAMM 1964, POŽGAJ *et al.* 1997, TREBULA – KLEMENT 2002, VIDELOV 2003, PERVAN 2009, DELIISKI – DZURENDA 2010, DELIISKI *et al.* 2010, 2013).

The aim of the present work is to suggest a mathematical description of the thermal conductivity of the wood above the hygroscopic range during freezing of both the free and the bound water in the wood. For achieving of this aim experimental data of different authors and the mathematical descriptions of the wood thermal conductivity during wood defrosting, made earlier by the first co-author (DELIISKI 1977, 2003, 2011, 2013) were used, with taking into account for the first time the influence of the fiber saturation point of wood species on the value of their  $\lambda$  during water freezing in the wood and the influence of the temperature on the fiber saturation point of the wood during its freezing.

## MATERIAL AND METHODS

### Mathematical description of the water freezing temperature in the wood

According to CHUDINOV (1966, 1968) and TOPGAARD – SÖDERMAN (2002), if the wood has a significant quantity of free water, i.e. if the cell holes and the gaps among the cells are almost completely filled with water, the centers of crystallization during the cooling arise in the water at temperatures around  $-5 \text{ °C} \div -6 \text{ °C}$ . If the wood moisture content is slightly larger than  $u_{fsp}$ , i.e. a small quantity of free water is present in the wood, then the centers of

crystallization in it arise at temperatures around  $-12\text{ }^{\circ}\text{C} \div -15\text{ }^{\circ}\text{C}$ . However, when there is only bound water in the wood, i.e.  $u \leq u_{\text{fsp}}$ , then the centers of crystallization arise at even lower temperatures. Besides this, CHUDINOV (1966) points out, that approximately  $0.12\text{ kg}\cdot\text{kg}^{-1}$  from the bound water in the wood remains in liquid state at extremely low temperatures on earth. TOPGAARD – SÖDERMAN (2002) experimentally establish at  $-24\text{ }^{\circ}\text{C}$  layers of unfrozen bound water in the wood with thickness of 1 nm.

Based on the results from personal experimental studies, CHUDINOV (1966, 1968) suggests a graph for the change of the temperature of freezing of the water in birch wood with fiber saturation point at  $T = 293.15\text{ K}$  (i.e. at  $t = 20\text{ }^{\circ}\text{C}$ )  $u_{\text{fsp}}^{293.15} = 0.3\text{ kg}\cdot\text{kg}^{-1}$  depending on  $u$ . According to this graph the water freezing temperature in the wood  $t_{\text{fr}}$  is equal to the following:  $t_{\text{fr}} \approx -20\text{ }^{\circ}\text{C}$  at  $u = 0.3\text{ kg}\cdot\text{kg}^{-1}$ ,  $t_{\text{fr}} \approx -10\text{ }^{\circ}\text{C}$  at  $u = 0.4\text{ kg}\cdot\text{kg}^{-1}$ ,  $t_{\text{fr}} \approx -7.5\text{ }^{\circ}\text{C}$  at  $u = 0.5\text{ kg}\cdot\text{kg}^{-1}$ ,  $t_{\text{fr}} \approx -6.5\text{ }^{\circ}\text{C}$  at  $u = 0.6\text{ kg}\cdot\text{kg}^{-1}$ , and for  $u \geq 0.8\text{ kg}\cdot\text{kg}^{-1}$ , the temperature  $t_{\text{fr}}$  asymptotically comes close to  $t_{\text{fr}} \approx -5\text{ }^{\circ}\text{C}$ . Due to the lack of other published data for the water freezing temperature in the wood, for the mathematical description of  $t_{\text{fr}}$  below, the quoted above data obtained by this author for the change of  $t_{\text{fr}}$  depending on  $u$  at  $u_{\text{fsp}}^{293.15} = 0.3\text{ kg}\cdot\text{kg}^{-1}$  has been used.

Taking into account the influence of  $u_{\text{fsp}}$  on  $t_{\text{fr}}$ , the shown in (CHUDINOV 1966, 1968) graphical dependency  $t_{\text{fr}}(u)$  can be approximated with the help of the following equation:

$$T_{\text{fr}} = 268.15 - 118.85 \exp[-9.9(0.3 + u - u_{\text{fsp}}^{293.15})^{1.3}] \text{ @ } 0.12\text{ kg}\cdot\text{kg}^{-1} \leq u \leq u_{\text{max}} . \quad (1)$$

With the help of the expression  $(0.3 + u - u_{\text{fsp}}^{293.15})$  in equation (1) the determined by CHUDINOV (1966) relationship's character of the influence of  $u$  on  $T_{\text{fr}}$  for birch wood with  $u_{\text{fsp}}^{293.15} = 0.30\text{ kg}\cdot\text{kg}^{-1}$  is accepted as valid for all wood species taking into account the concrete value of their  $u_{\text{fsp}}^{293.15}$ . It is only possible to make a future clarification of equation (1) when having extensive experimental data for the change in  $T_{\text{fr}}$  depending on  $u$  for wood species with different value of  $u_{\text{fsp}}^{293.15}$ .

The calculated according to equation (1) change in  $t_{\text{fr}}$  for birch wood with  $u_{\text{fsp}}^{293.15} = 0.30\text{ kg}\cdot\text{kg}^{-1}$ , beech wood with  $u_{\text{fsp}}^{293.15} = 0.31\text{ kg}\cdot\text{kg}^{-1}$ , and poplar wood with  $u_{\text{fsp}}^{293.15} = 0.35$  (NIKOLOV – VIDELOV 1987, DELIISKI 2003, 2011, 2013, DELIISKI – DZURENDA 2010) depending on  $u$  in the range  $0.2\text{ kg}\cdot\text{kg}^{-1} \leq u \leq 1.0\text{ kg}\cdot\text{kg}^{-1}$  is shown on Fig. 1. The graph for birch wood shown on this figure coincides completely with the suggested by CHUDINOV (1966, 1968) graph.

The equation (1) is used below for the determination of  $T_{\text{fr}}$  in the mathematical description of the fiber saturation point and the wood thermal conductivity during the freezing of both the free and the bound water in the wood.

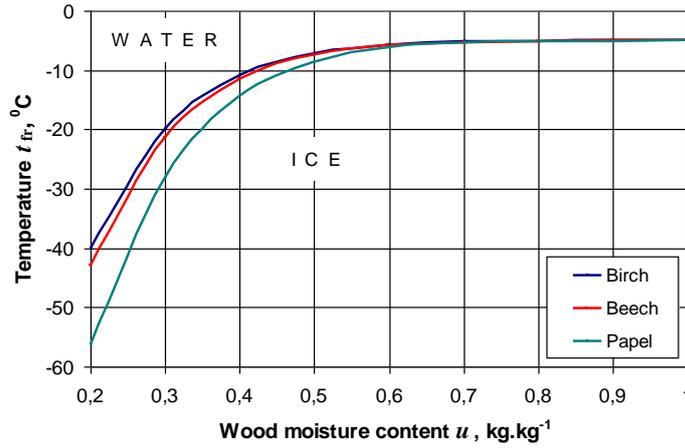


Fig. 1 Change in  $t_{fr}$  for birch, beech and poplar wood, depending on moisture content  $u$ .

### Influence of the temperature on the fiber saturation point

Based on the results of wide experimental investigations, A. J. STAMM (1964) suggests the following equation, which reflects the influence of the temperature on the fiber saturation point of the non-frozen wood:

$$u_{fsp} = u_{fsp}^{293.15} - 0.001(T - 293.15). \quad (2)$$

In the specialized literature there are too few records concerning the influence of  $T$  on  $u_{fsp}$  of wood, which contains ice. KÜBLER *et al.* (1973) noted that below 0 °C the bound water diffused out of the cells' walls and crystallized as ice in the cells' cavities even when free water was present. The formation of ice from diffused bound water causes significant swelling of the frozen wood (KÜBLER 1962, SHMULSKY – SHVETS 2006).

A proportional reduction of the maximum amount of bound water that cell's walls can hold at freezing temperatures have been experimentally determined by SHMULSKY – SHVETS (2006). CHUDINOV (1966, 1968) also points out that with a decrease in  $T$ , it is expected that  $u_{fsp}$  of wood, which contains ice also decreases, because the portion of bound water, which is transformed into ice, stops being bound and becomes free.

Since equation (2) is generally accepted in the literature, then during the mathematical description of  $\lambda$  below, this equation is used for the reflection of the influence of  $T$  on  $u_{fsp}$  for wood at  $t < 0$  °C before the freezing of the free and bound water in it, i.e:

$$u_{fsp} = u_{fsp}^{293.15} - 0.001(T - 293.15) \quad @ \quad T > T_{fr}. \quad (3)$$

After the freezing of the free and during the freezing of the bound water in the wood, a constant value of  $u_{fsp}$  is used in the mathematical description of  $\lambda$ , which the wood has at the temperature of the beginning of its freezing  $T_{fr}$ , i. e.:

$$u_{fsp} = u_{fsp}^{293.15} - 0.001(T_{fr} - 293.15) \quad @ \quad T \leq T_{fr}. \quad (4)$$

Due to the lack of published data on the influence of  $t$  on  $u_{fsp}$  for wood containing ice, in the derived below mathematical description of  $\lambda$  at  $T < T_{fr}$  constant values of  $u_{fsp}$  are obtained for each wood specie according to equation (4).

## Mathematical description of the wood thermal conductivity during freezing of the water in the wood

The mathematical description of the wood thermal conductivity  $\lambda$  during wood freezing has been done using the experimentally determined in the dissertations by KANTER (1955) and CHUDINOV (1966, 1968) data for its change as a function of  $t$  and  $u$ . This data for  $\lambda(t, u)$  finds a wide use in both the European (SHUBIN 1990, TREBULA – KLEMENT 2002, VIDELOV 2003) and the American specialized literature (STEINHAGEN 1986, 1991, KHATTABY – STEINHAGEN 1992, 1993) when calculating various processes of the thermal processing of frozen and non-frozen wood.

The approach used in the mathematical description of  $\lambda$  is analogous to this, which has been used earlier in the description of  $\lambda(t, u, \rho_b)$  of non-frozen wood (DELIISKI 1977) and of frozen and non-frozen wood without taking into account the influence of the fiber saturation point of wood species on the value of its  $\lambda$  and the influence of the temperature on the fiber saturation point (DELIISKI 2003). The description of  $\lambda(t, u, \rho_b)$  given in (DELIISKI 1977) has been used later for different computations of non-stationary temperature distribution in subjected to thermal treatment prisms in the veneer production (DELIISKI 1979) and in wooden beams (DELIISKI 1988, OLEK – GUZENDA 1995, etc.).

The thermal conductivity of the wood during its freezing can be calculated with the help of the following equations for  $\lambda(T, u, \rho_b, u_{fsp})$ :

$$\lambda = \lambda_0 \gamma [1 + \beta(T - 273.15)], \quad (5)$$

$$\lambda_0 = K_{ad} v [0.165 + (1.39 + 3.8u) \cdot (3.3 \cdot 10^{-7} \rho_b^2 + 1.015 \cdot 10^{-3} \rho_b)], \quad (6)$$

$$v = 0.15 - 0.07u \quad @ \quad u \leq u_{fsp} + 0.1 \text{ kg} \cdot \text{kg}^{-1}, \quad (7)$$

$$v = 0.1284 - 0.013u \quad @ \quad u > u_{fsp} + 0.1 \text{ kg} \cdot \text{kg}^{-1}. \quad (8)$$

The equations, which have been suggested by CHUDINOV (1966, 1968) and shown in (DELIISKI 1977, 2003) can be used for the determination of the values of the coefficient  $K_{ad}$  in equation (6), which takes into account the influence on  $\lambda_0$  of the heat flux towards the separate anatomic directions of the wood, i.e. for the determination of  $\lambda_r$ ,  $\lambda_t$  and  $\lambda_p$ . In DELIISKI (2003) more precise values of  $K_{ad}$  for 10 wood species have been determined. For the discussed in this paper beech and poplar wood the following values of  $K_{ad}$  in radial direction have been determined: for beech wood  $K_r = 1.35$  and for poplar wood  $K_r = 1.48$ .

The coefficients  $\gamma$  and  $\beta$  in equation (5) are calculated by the following equations:

- For non-frozen wood, i.e. when  $T_{fr} < T \leq 273.15 \text{ K}$  and simultaneously with this  $u > u_{nfw}$  or when  $u \leq u_{nfw}$  and simultaneously with this  $273.15 \text{ K} \leq T \leq 273.15 \text{ K}$ :

$$\gamma = 1.0, \quad (9)$$

$$\beta = (2.05 + 4u) \cdot \left( \frac{579}{\rho_y} - 0.124 \right) \cdot 10^{-3} \quad @ \quad u \leq u_{fsp} + 0.1 \text{ kg} \cdot \text{kg}^{-1} \quad (10)$$

$$\beta = 3.65 \left( \frac{579}{\rho_y} - 0.124 \right) \cdot 10^{-3} \quad @ \quad u > u_{fsp} + 0.1 \text{ kg} \cdot \text{kg}^{-1}. \quad (11)$$

- For frozen wood, i.e. when  $213.15 \text{ K} \leq T \leq T_{fr}$  and simultaneously with this  $u > u_{nfw}$ :

$$\gamma = 1 + 0.34[1.15(u - u_{fsp})], \quad (12)$$

$$\beta = 0.002(u - u_{fsp}) - 0.0038 \left( \frac{579}{\rho_b} - 0.124 \right). \quad (13)$$

The values of  $u_{fsp}$  in equations above are calculated with the help of the equations (3) and (4). The value of the non-frozen water in the wood at given temperature  $T$  can be calculated according to following equation:

$$u_{nfw} = 0.12 + (u_{fsp} - 0.12) \exp[0.0567(T - 268.15)] \quad @ \quad 213.15 \text{ K} \leq T \leq T_{fr}. \quad (14)$$

For the calculation of  $\lambda$  in the cases, when during the wood freezing  $T \leq T_{fr}$  and simultaneously with this  $u_{nfw} \leq u < u_{max}$ , the value of  $\lambda_{fr}$  must be initially determined by substituting  $T = T_{fr}$  in equation (5) as well as the values of  $\gamma$  from equation (9) and of  $\beta$  from equation (10). After this, with the usage of the coefficient  $\beta$  from equation (13), the values of  $\lambda$  are calculated with the help of the following equation:

$$\lambda = \lambda_{fr}[1 + \beta(T - T_{fr})] \quad @ \quad T \leq T_{fr} \quad \& \quad u_{nfw} \leq u \leq u_{max}. \quad (15)$$

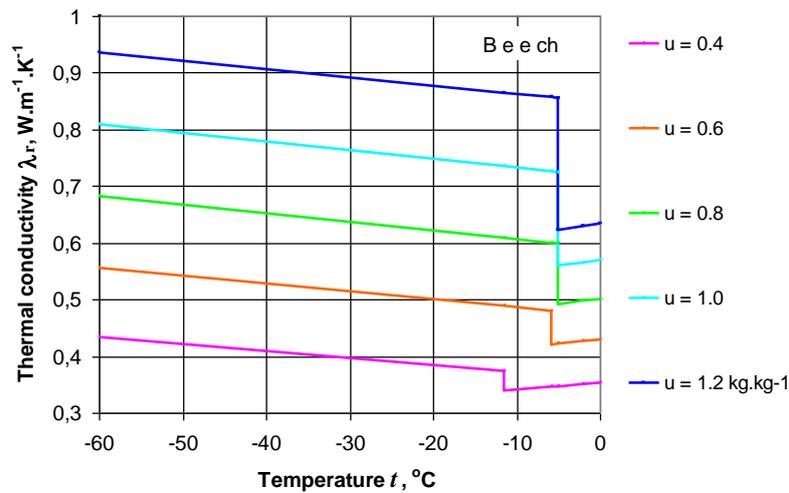
## RESULTS AND DISCUSSION

For the computation of  $\lambda$  according to equations (1) ÷ (15) a software program has been prepared in FORTRAN (DORN – MCCracken 1972), which has been input in the calculation environment of Visual Fortran Professional.

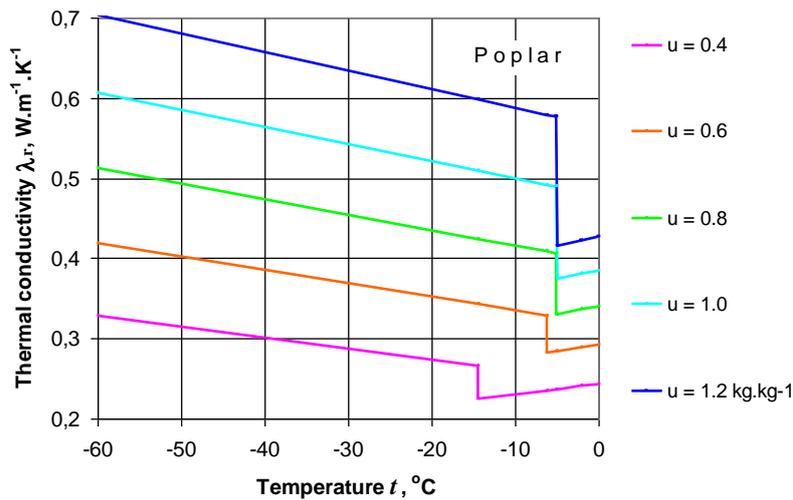
With the help of the program computations have been made for the determination of  $\lambda$  in the ranges  $0.4 \text{ kg} \cdot \text{kg}^{-1} \leq u \leq 1.2 \text{ kg} \cdot \text{kg}^{-1}$  and  $213.15 \text{ K} \leq T \leq 273.15 \text{ K}$ , i.e.  $-60 \text{ }^\circ\text{C} \leq t \leq 0 \text{ }^\circ\text{C}$ . As an example, the thermal conductivity in the radial direction of often used in the veneer and plywood production beech (*Fagus Silvatica* L.) and poplar (*Populus alba* L.) wood above the hygroscopic range has been calculated below. The linear character of the dependences  $\lambda_r(t)$  allows for the calculation of the change of  $\lambda$  to start from temperature  $-60 \text{ }^\circ\text{C}$ , even though the lowest temperature of the experimental data for  $\lambda$  used in the mathematical description of  $\lambda$  was  $-40 \text{ }^\circ\text{C}$ .

During the computations of  $\lambda_r(t, u)$  the values of  $\rho_b = 560 \text{ kg} \cdot \text{m}^{-3}$  and  $u_{fsp}^{293.15} = 0.31 \text{ kg} \cdot \text{kg}^{-1}$  for the beech wood and  $\rho_b = 355 \text{ kg} \cdot \text{m}^{-3}$  and  $u_{fsp}^{293.15} = 0.35 \text{ kg} \cdot \text{kg}^{-1}$  for the poplar wood were used (NIKOLOV – VIDELOV 1987, DELIISKI – DZURENDA 2010).

On Fig. 2 and Fig. 3 the calculated according to equations (1) ÷ (15) change in the thermal conductivity of beech and poplar wood respectively during the freezing of the both the free and the bound water in the wood, depending on  $t$  and  $u$  is shown.



**Fig. 2** Change in  $\lambda_r$  of beech wood during freezing of the free and the bound water in the wood, depending on  $t$  and  $u > u_{fsp}$ .



**Fig. 3** Change in  $\lambda_r$  of poplar wood during freezing of the free and the bound water in the wood, depending on  $t$  and  $u > u_{fsp}$ .

On the graphs of Fig. 2 and Fig. 3 it can be seen that the decrease in  $t$  at a given value for  $u$  leads to an increase in  $\lambda$  for wood containing ice as a consequence of the increase in the thermal conductivity of the ice with the decrease of  $t$ , and to decrease in  $\lambda$  for wood, which does not contain ice.

The change in  $\lambda$  of frozen and non-frozen wood depending on  $t$ , calculated according to equation (5) is linear. From the analysis on Fig. 2 and Fig. 3 it can also be seen that at a given value of  $t$  an increase in  $u$  for wood both not containing and containing ice, causes almost proportional increase in  $\lambda$ .

On the graphs of Fig. 2 and Fig. 3 it can also be seen that at  $t = t_{fr}$  jumps take place in  $\lambda$  for wood with given  $u > u_{fsp}$  from the lower values of  $\lambda$  of the non-frozen wood to the larger values of  $\lambda$  of the frozen wood. These jumps are explained by the phase transition into ice of the extra cooled part of the free water in the wood at  $t = t_{fr}$  during wood cooling.

Namely, at the values of  $t = t_{fr}$  the influence on  $\lambda$  of a significant difference in the thermal conductivity of the water in a liquid and hard aggregate state occurs.

Because of the increase of the ice quantity from the free water in the wood with increasing of  $u$  and also because of the increasing at  $t_{fr}$  of the extra cooled part of free water with increasing of  $u$ , the change  $\Delta\lambda$  in the thermal conductivity during these jumps increases non-linearly depending on  $u$ : from  $\Delta\lambda = 0.035 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $u = 0.4 \text{ kg}\cdot\text{kg}^{-1}$  and  $t_{fr} = -11.47 \text{ }^\circ\text{C}$  to  $\Delta\lambda = 0.232 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $u = 1.2 \text{ kg}\cdot\text{kg}^{-1}$  and  $t_{fr} = -5 \text{ }^\circ\text{C}$  for the beech wood and from  $\Delta\lambda = 0.041 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $u = 0.4 \text{ kg}\cdot\text{kg}^{-1}$  and  $t_{fr} = -14.49 \text{ }^\circ\text{C}$  to  $\Delta\lambda = 0.161 \text{ W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$  at  $u = 1.2 \text{ kg}\cdot\text{kg}^{-1}$  and  $t_{fr} = -5 \text{ }^\circ\text{C}$  for poplar wood.

The larger value of the basic density  $\rho_b = 560 \text{ kg}\cdot\text{m}^{-3}$  of beech wood in comparison with  $\rho_b = 355 \text{ kg}\cdot\text{m}^{-3}$  of poplar wood causes larger values of  $\lambda_r$  of the beech wood in comparison with  $\lambda_r$  of the poplar wood at given values of  $t$  and  $u$ , independent of the fact that  $K_r = 1.48$  for poplar wood is larger than  $K_r = 1.35$  for beech wood. In equation (6)  $\rho_b$  and  $K_r$  participate as multipliers for the calculation of  $\lambda_{or}$ , and while  $K_r$  participate only in its first degree,  $\rho_b$  participates both in its first and second degrees.

## CONCLUSIONS

The present paper presents the mathematical description of the wood thermal conductivity of frozen and non-frozen wood above the hygroscopic range, which takes into account to a maximum degree the physics of the process of freezing of both the free and the bound water in the wood. It reflects the influence of the temperature, wood moisture content, and wood density and for the first time the influence of the fiber saturation point of wood species on the value of their  $\lambda$  during water freezing in the wood and the influence of the temperature on the fiber saturation point of the wood during its freezing.

An equation for the determination of the temperature of the beginning of the freezing of the water in the wood,  $t_{fr}$ , has been derived depending on the wood moisture content  $u$  and on the standardized fiber saturation point of the wood at temperature  $T = 293.15 \text{ K}$ , i.e. at  $t = 20 \text{ }^\circ\text{C}$ . For the computation of the thermal conductivity of the wood during its freezing, equations have been presented as well.

For the computation of the wood thermal conductivity during the freezing of the wood according to the suggested mathematical description of  $\lambda$  a software program has been prepared in FORTRAN, which has been input in the calculation environment of Visual Fortran Professional. With the help of the program computations have been carried out for determination of the thermal conductivity in the radial direction of beech and poplar wood with moisture content  $0.4 \text{ kg}\cdot\text{kg}^{-1} \leq u \leq 1.2 \text{ kg}\cdot\text{kg}^{-1}$  at a temperature range from  $213.15 \text{ K}$  to  $273.15 \text{ K}$ , i.e. from  $-60 \text{ }^\circ\text{C}$  to  $0 \text{ }^\circ\text{C}$ .

The obtained results show that a decrease in  $t$  at a given value for  $u$  leads to an increase in  $\lambda$  for wood containing ice as a consequence of the increase in the thermal conductivity of the ice with the decrease of  $t$ , and to a decrease in  $\lambda$  for wood, which does not contain ice. The change in  $\lambda$  depending on  $t$  is linear. The results show also that at a given value of  $t$  an increase in  $u$  both for non-frozen wood and for wood containing ice, causes almost proportional increase in  $\lambda$ .

If non-frozen wood with given  $u > u_{fsp}$  is subjected to cooling and the wood temperature decreases and reaches a temperature  $t = t_{fr}$ , a jump take place in  $\lambda$  from the

lower value of  $\lambda$  of the non-frozen wood to the larger value of  $\lambda$  of the frozen wood. This jump is explained by the phase transition into ice of the extra cooled part of the free water in the wood at  $t = t_{fr}$  during wood cooling.

The larger value of the basic density  $\rho_b = 560 \text{ kg}\cdot\text{m}^{-3}$  of beech wood in comparison with  $\rho_b = 355 \text{ kg}\cdot\text{m}^{-3}$  of poplar wood causes larger values of  $\lambda_r$  of the beech wood in comparison with  $\lambda_r$  of the poplar wood at given values of  $t$  and  $u$ ,

The obtained results can be used for mathematical modeling of the wood freezing process and for technological analysis of processes of thermal and hydrothermal treatment of frozen wood materials, as well as in software of systems for model based automatic control (HADJISKI 2003) of such treatment.

### **Symbols:**

$T$  – temperature [K]:  $T = t + 273.15$ ,  
 $t$  – temperature [°C]:  $t = T - 273.15$ ,  
 $u$  – moisture content [ $\text{kg}\cdot\text{kg}^{-1} = \%/100$ ],  
 $\lambda$  – thermal conductivity [ $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$ ],  
 $\rho$  – density [ $\text{kg}\cdot\text{m}^{-3}$ ],  
 $\Delta$  – difference (for the change in  $\lambda$ ),  
& – and simultaneously with this,  
@ – at.

### **Subscripts and superscripts:**

ad – anatomical direction,  
b – basic (for wood density, based on dry mass divided to green volume),  
fr – freezing,  
fsp – fiber saturation point,  
max – maximum possible value,  
nfw – non-frozen water,  
0 – initial (at 0 °C for  $\lambda$ ),  
p – parallel to the fibers,  
r – radial direction,  
t – tangential direction,  
293.15 – at 293.15 K, i.e. at 20 °C.

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