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FUNDAMENTALS OF CALCULATION OF ELEMENTS FROM SOLID AND GLUED TIMBER WITH REPEATED OBLIQUE TRANSVERSE BENDING, TAKING INTO ACCOUNT THE CRITERION OF DEFORMATION

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ABSTRACT

The actual use of solid or glued timber elements that are in direct or oblique transverse bending under single or repeated loads is not considered in the current standards. In particular, the formation of folds in the compressed zone of pure bending and obtained a significant margin of safety. Therefore, the concept of "calculated cross-section" is proposed in the article. It describes the distribution of deformations in the height of the cross-section and, also, the layer-by-layer connection between deformations and stresses in a section of bending timber elements by considering the formation of folds in the compressed zone under one-time short-term or repeated loads. Therefore, the method for calculating wooden beams from solid and glued timber using a deformation model was developed. The formation of folds in the compressed zone is predicted by the method of the pure bending zone by analyzing the distribution of stresses on height in the compressed and stretched zones of the calculated cross-section.

Key words: solid and glued timber, rigidity, bearing capacity

INTRODUCTION

Timber is used in many sectors of the world economy, including construction and civil engineering (PINCHEVSKA *et al.* 2019; HOMON *et al.* 2022; SUBRAMMANIAN 2010; YASNIY *et al.* 2022; WDOWIAK-POSTULAK 2020). This article considers the operation of load-bearing elements and structures from timber. The most common for timber elements is its bending load (ANSHARI *et al.* 2017; GOMON *et al.* 2022; DE LA ROSA GARSIA *et al.* 2019; BETTS *et al.* 2010; VAHEDIAN *et al.* 2019). A lot of work is devoted to solving the problem of bending a wooden element, because from the point of view of a practicing engineer, this work is the most interesting. Especially this is important for solid and glued timber (DONADON *et al.* 2020; GOMON *et al.* 2020; SOBCZAK-PIASTKA *et al.* 2020; SORIANO *et al.* 2016). Since even when working with a beam with a zone of pure bending, there are two types of layers (fibers): some of which work at compressive stresses of different intensities, and others – at tensile stresses.

In the calculated cross-sectional model, when determining the bearing capacity of solid and glued timber structures, a triangular stress diagram in compressed and stretched areas of timber under the action of single loads is adopted. In such plots, the height of the compressed zone of timber and the position of the neutral plane (neutral line) does not change during increasing loads.

The solution of problems to determine the bearing capacity of timber elements in normal sections is based on the concept of strength criteria. Currently, in the norms (EUROCODE 5: 1995; DBN B.2.6-161: 2017) there is only one criterion, when in the normal cross-section of the stress across the entire cross-section or at the farthest point of the compressed or stretched zone of timber $\sigma_{m,d} = f_{m,d}$. The expression for the criterion of strength can be represented for force effects in general dependence

$$\sigma_{m,d} = f_{m,d} = const \tag{1}$$

Normal stresses $\sigma_{m,d}$ determined by the known formula of resistance of materials

$$\sigma_{m,d} = \frac{M_d}{W_d} \tag{2}$$

The neutral plane between the compressed and stretched zones (neutral line in the calculated cross-section) already at the initial stages of loading with increasing bending moment begins to gradually shift towards the stretched zone (GOMON and PAVLUK 2017), and with oblique bending – with a slight turn to one side. In the ultimate state, the compressed zone of the bending element of timber occupies about 60% of the calculated cross-section, or, stretched about 40% due to the anisotropy of timber because the strength of timber in compression along the fibers is two times less than the tensile strength of timber. But in Formula 2 uses the moment of resistance of the cross-section, for example for a rectangular cross-section in the form

$$W_d = \frac{bh^2}{6} \tag{3}$$

which in this form is determined by the location of the position of the neutral line passing through the center of gravity of the cross-section at a height h/2 and is unchanged from boot to crash. This leads to the fact that in experimental studies in the compressed zone of the bending element of solid or glued timber, we obtain false values of ultimate strength for the most distant fibers, which exceeds 1.5 times the actual values of timber strength to central compression and accordingly underestimated strength values of the most remote wood fibers in the stretched zone, which reach only 70-75% of the values of ultimate tensile strength. With an increase in the bending moment in the element of timber in the extreme fibers of the compressed zone in the pre-destructive state of deformation reaches critical values (GOMON *et al.* 2019), begins to form and spread deep into the folded element, which is difficult to see and this leads to redistribution of internal stresses in the calculated cross-section and almost instantaneous destruction of the most stressed fibers of the stretched zone.

When calculating the structures of timber for oblique transverse bending according to the current rules (EUROCODE 5: 1995; DBN B.2.6-161: 2017) does not provide for the operation of such elements with variable low-cycle re-loads (GOMON and PAVLUK 2017; GOMON *et al.* 2019) (using a cross-section of dense solid wood). For the developed technique the concept of "calculated cross-section" is used (GOMON *et al.* 2019), which provides for the formation of folds in the pre-destructive state and four stages of stress-strain state of a wooden element (GOMON and PAVLUK 2017; GOMON *et al.* 2019) working on direct or oblique transverse bending.

The aim was to investigate the operation of glued timber beams under the action of repeated loads with oblique transverse pure bending and to develop a deformation method for their calculation.

RESULTS AND DISCUSSION

The developed method of calculation of wooden beams of the rectangular section according to the deformation model is based on the following preconditions:

1. the normal section of a longitudinal axis of an element in the centre of the span is calculate;
2. stresses in the normal cross-section were calculated using two functions $\sigma_{t,d}$ and $\sigma_{c,d}$:

a) the first function is rectilinear and describes the stress in the stretched zone of the wooden element and operates within 0 and y_t

$$\sigma_{t,d} = f_1(u) = E_{0,05} \cdot u_{t,d} \,, \tag{4}$$

Where: $E_{0,05}$ - modulus of elasticity of timber under tensile action; $u_{t,d}$ - relative deformations under the action of tensile wood;

b) the second function describes the stresses that occur in two parts of the compressed zone of the wooden element within 0 and y_c

$$\sigma_{c,d} = f_2(u) = k_1 u_{c,d} + k_c u_{c,d}^2 \,, \tag{5}$$

Where: $f_1(u)$ - the stress distribution function of the stretched zone; $f_2(u)$ - the stress distribution function of the compressed zone; $u_{c,d}$ - relative deformations during compression of timber in the calculated cross-section; k_1, k_c - the coefficients of the polynomial are proposed to be determined by expressions:

$$k_1 = \frac{2\, f_{c,o,d}}{u_{c,fin,d}}; \quad k_c = -\frac{f_{c,o,d}}{u_{c,fin,d}^2}, \tag{6}$$

Where: $f_{c,0,d}$ the estimated value of compression along the fibers; $u_{c,fin,d}$ - relative complete deformations during timber compression.

3. elements are considered in which the influence of the transverse force on the deflections is not significant;
4. average values of temporary resistance of wooden elements are accepted as calculated;
5. deformations of the compressed zone are accepted with a minus sign, stretched - with a plus sign.

The criterion for loss of load-bearing capacity of the section is taken (GOMON and PAVLUK 2017; GOMON *et al.* 2019):

- destruction of the stretched zone of timber at achievement by the most stretched layer of limit values of deformations;
- loss of balance between internal and external efforts - an extreme criterion.

For direct and oblique bending under repeated loads, the calculated diagram of the physical state of timber for the compressed zone in the calculated cross-section of the element of solid and glued timber is taken as curvilinear, which corresponds more to the actual work of timber cross-sections for work, both compression and transverse bend. This diagram is described by a polynomial of the second degree (5) taking into account the descending branch, and the stress distribution along the height of the stretched zone on the n-th cycle of load application is assumed to be linear (Fig. 1).

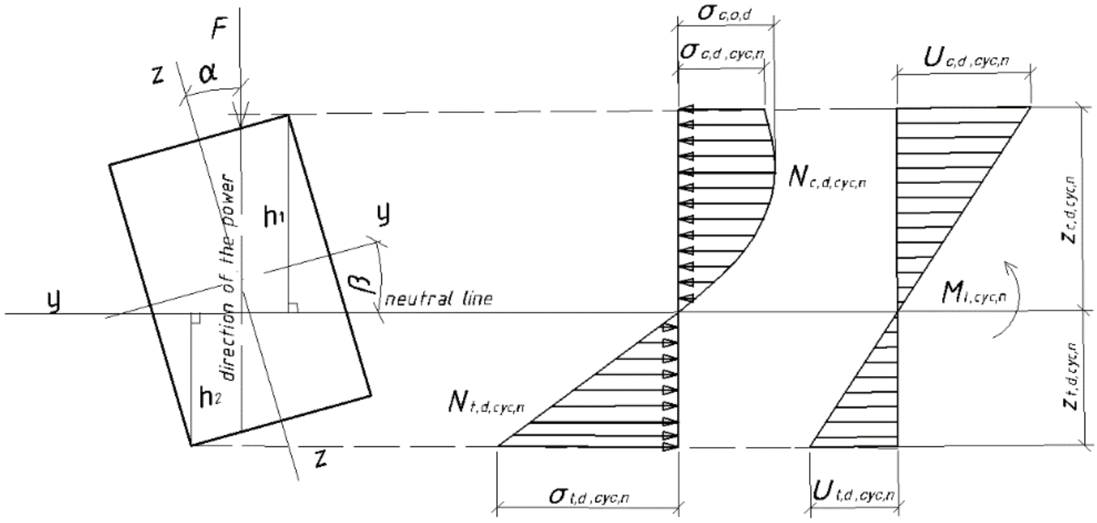

Fig.1. Distribution of the greatest stresses and deformations in the normal section of a beam for works on a pure oblique bend:

$u_{c,d,cyc,n}$ - relative compression deformations in the extreme timber fiber of the element zone on the n - cycle; $u_{t,d,cyc,n}$ - relative tensile deformations in the extreme timber fiber of the element zone on the n - cycle; $f_{c,0,d}$ - calculated stresses of the fiber of the compressed zone of the element on the n - cycle; $\sigma_{c,d,cyc,n}$ - the stress of the most distant fiber of the compressed zone of the element on the n - cycle; $\sigma_{t,d,cyc,n}$ - tensile stress in the extreme timber fiber of the stretched zone of the element on the n - cycle; y-y, z-z - the main axes of the cross-section of the element; α the angle of the external load F on the element relative to the main axis z-z; z_t - height of the stretched cross-sectional area; $z_{c,d,cyc,n}$ - the height of the compressed cross-sectional area; $M_{1,cyc,n}$ - bending moment from external load; $N_{c,d,cyc,n}$ - equivalent of internal forces in compressed timber of normal design cross-section; $N_{t,d,cyc,n}$ - the equivalent of internal forces in stretched timber of normal design cross-section.

Deformations in the calculated cross-section are determined through the curvature at any point of the section and taking into account the smallness of their values. Then the deformation on *n*-th load application cycles is calculated by expressions as follows

$$u_{c,d,cyc,n} = z_c \frac{1}{\rho_{cyc}}; \qquad u_{t,d,cyc,n} = z_t \frac{1}{\rho_{cyc}} \tag{7}$$

Where: $\frac{1}{\rho_{cyc}}$ - the curvature of the element on the n-th cycle of repeated loads; z_c – the distance from the neutral line to the point of determination of relative compression strains; z_t – the distance from the neutral line to the point of determination of relative tensile deformations.

Stresses in the normal cross section of the beam were calculated using functions $\sigma_{t,d}$ and $\sigma_{c,d}$, taking into account certain transformations: the first function that describes the stresses that occur in two sections of the compressed.

Zone of normal section of a wooden element with oblique bending in the range from 0 to the n-th cycle of loading is expressed by a polynomial of the second degree in the form

$$\sigma_{c,d,cyc,n} = f_1(u) = k_1 u_{c,d,cyc,n} + k_c u^2_{c,d,cyc,n} = k_1 \frac{1}{\rho_{cyc}} z_{c,cyc,n} + k_2 (\frac{1}{\rho_{cyc}})^2 z_{c,cyc,n} \tag{8}$$

Where: $u_{c,cyc,n}$ - relative deformations of timber compression in the element under the action of loading on the n-th cycle; $\frac{1}{\rho_{cyc}}$ - the curvature of the element on the n-th cycle of repeated loads.

$$\sigma_{t,d,cyc,n} = f_2(u) = E \cdot u_{t,d,cyc,n} = E(\frac{1}{\rho_{cyc}})z_{t,cyc,n} \tag{9}$$

Where: E - modulus of elasticity of timber under tensile action; - relative tensile deformations of the timber in the element under the action of applying a load on the n-th cycle; $\frac{1}{\rho_{cyc}}$ curvature of the element on the n– th cycle of repeated loads.

Equilibrium equations for the cross-section (Fig. 1) have the form

$$\sum M_{н,л,cyc,n} = 0; M_{cyc,n} = M_{c,d,cyc,n} + M_{t,d,cyc,n} \tag{10}$$

$$\sum N_{cyc,n} = 0; N_{c,d,cyc,n} = N_{t,d,cyc,n} \tag{11}$$

Where: $M_{cyc,n}$, $M_{c,d,cyc,n}$ and $M_{t,d,cyc,n}$ – bending moments according to external load, forces in compressed and stretched timber on the n-th loading cycle; $N_{c,d,cyc,n}$ and $N_{t,d,cyc,n}$ - equivalent internal forces in compressed and stretched timber of normal design cross-section, respectively.

Based on the deformations (Fig. 1), the stress in the normal section is described by two functions $\sigma_{c,d,cyc,n}$ and $\sigma_{t,d,cyc,n}$ in three different sections: the first section is the tensile section from the bottom of the element to the neutral line; the second section - from the neutral line to the maximum voltage in the compressed zone; the third section - from the maximum stress in the compressed zone to the top of the element. Taking into account functions $\sigma_{c,d,cyc,n}$ and $\sigma_{t,d,cyc,n}$, the coefficients of polynomials k_1 and k_c, as well as the equilibrium equations $M_{cyc,n}$ and $N_{c,d,cyc,n}$ of the compressive force in the normally calculated cross-section of the bending element are defined as the sum of forces arising in the compressed zone cross-section, which consists of two sections: the first in the form of a parallelogram; the second in the form of a triangle

$$N_{c,d,cyc,n} = N_{1c,d,cyc,n} + N_{2c,d,cyc,n}, \tag{12}$$

Where: $N_{1c,d,cyc,n}, N_{2c,d,cyc,n}$ – compression forces in the bending element with oblique bending in different sections on the n-th cycle of the application of repeated loads equal to:

on the first section

$$N_{1c,d,cyc,n} = \int_0^{z_{1c,cyc,n}} f_1(u_{c,d,cyc,n})dA = b\int_0^{z_{1c,cyc,n}} f_1(u_{c,d,cyc,n})dz = b\int_0^{z_{1c,cyc,n}} \left(k_1(\frac{1}{\rho_{cyc}})z_{1c,cyc,n} + k_c(\frac{1}{\rho_{cyc}})^2 z_{1c,cyc,n}^2 \right) dz =$$
$$= b\left(k_1(\frac{1}{\rho_{cyc}})\frac{z_{1c,cyc,n}^2}{2} + k_c(\frac{1}{\rho_{cyc}})^2 \frac{z_{1c,cyc,n}^3}{3} \right), \tag{13}$$

on the second section

$$N_{2c,d,cyc,n} = \int_{z_{1c,cyc,n}}^{z_{2c,cyc,n}} f_1(u_{c,d,cyc,n})dA = b\int_{z_{1c,cyc,n}}^{z_{1c,cyc,n}} f_1(u_{c,d,cyc,n})f(b)dz =$$

$$= \int_{1c,cyc,n}^{z_{2c,cyc,n}} \left((k_1(\frac{1}{\rho_{cyc}})z_{2c,cyc,n} + k_c(\frac{1}{\rho_{cyc}})^2 z^2_{2c,cyc,n}) \cdot (a_1 z_{2c,cyc,n} + a_2 \right) dz =$$

$$= \int_{1c,cyc,n}^{2c,cyc,n} \left(k_1(\frac{1}{\rho_{cyc}})a_1 z^2_{2c,cyc,n} + k_1(\frac{1}{\rho_{cyc}})a_2 z_{2c,cyc,n} + k_c(\frac{1}{\rho_{cyc}})^2 a_1 z^3_{2c,cyc,n} + k_c(\frac{1}{\rho_{cyc}})^2 a_2 z^2_{2c,cyc,n} \right) dz = \quad (14)$$

$$= k_1(\frac{1}{\rho_{cyc}})a_1 \frac{z^3_{2c,cyc,n}}{3} + k_1(\frac{1}{\rho_{cyc}})a_2 \frac{z^2_{2c,cyc,n}}{2} + k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^4_{2c,cyc,n}}{4} + k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^3_{2c,cyc,n}}{3} -$$

$$- k_1(\frac{1}{\rho_{cyc}})a_1 \frac{z^3_{1c,cyc,n}}{3} - k_1(\frac{1}{\rho_{cyc}})a_2 \frac{z^2_{1c,cyc,n}}{2} - k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^4_{1c,cyc,n}}{4} - k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^3_{1c,cyc,n}}{3},$$

Where: $b-$ the width of the cross section of the beam $dA = f(b)dz$;

$$f(b) = a_1 z_{c,cyc,n} + a_2;\ a_1 = \frac{b}{z_{1c,cyc,n} - z_{2c,cyc,n}};\ a_2 = \frac{b \cdot z_{2c,cyc,n}}{z_{1c,cyc,n} - z_{2c,cyc,n}} \quad (15)$$

Substituting formulas $N_{1c,d,cyc,n}$ and $N_{2c,d,cyc,n}$ into $N_{c,d,cyc,n}$ we obtain

$$N_{c,d,cyc,n} = b\left(k_1(\frac{1}{\rho_{cyc}})\frac{z^2_{1c,cyc,n}}{2} + k_c(\frac{1}{\rho})^2 \frac{z^3_{1c,cyc,n}}{3} \right) + k_1(\frac{1}{\rho_{cyc}})a_1 \frac{z^3_{2c,cyc,n}}{3} + k_1(\frac{1}{\rho_{cyc}})a_2 \frac{z^2_{2c,cyc,n}}{2} +$$

$$+ k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^4_{2c,cyc,n}}{4} + k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^3_{2c,cyc,n}}{3} - k_1(\frac{1}{\rho_{cyc}})a_1 \frac{z^3_{1c,cyc,n}}{3} - k_1(\frac{1}{\rho_{cyc}})a_2 \frac{z^2_{1c,cyc,n}}{2} - \quad (16)$$

$$- k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^4_{1c,cyc,n}}{4} - k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^3_{1c,cyc,n}}{3}.$$

Tensile force in the stretched zone of the normal section of the bending element, which consists of two sections: the first in the form of a parallelogram; the second in the form of a triangle on the n-th cycle of the application of repeated loads is determined

$$N_{t,d,cyc,n} = N_{1t,d,cyc,n} + N_{2t,d,cyc,n}, \quad (17)$$

Where: $N_{1t,d,cyc,n}, N_{2t,d,cyc,n}$ - tensile forces in the bending element with oblique bending in different sections on the n-th cycle of the application of repeated loads equal to:

on the first section

$$N_{1t,d,cyc,n} = \int_0^{z_{1t,cyc,n}} f_2(u_{t,d,cyc,n})dA = \int_0^{z_{1t,cyc,n}} f_2(u_{t,d,cyc,n})bdz = \int_0^{z_{1t,cyc,n}} E_t(\frac{1}{\rho_{cyc}})z_{1t,cyc,n}bdz =$$

$$= E_t(\frac{1}{\rho_{cyc}})b\frac{z^2_{1t,cyc,n}}{2}, \quad (18)$$

on the second section

$$N_{2t,d,cyc,n} = \int_{z_{1t,cyc,n}}^{z_{2t,cyc,n}} f_2(u_{t,d,cyc,n})dA = \int_{z_{1t,cyc,n}}^{z_{2t,cyc,n}} f_2(u_{t,d,cyc,n}) \cdot f(b)dz = \int_{z_{1t,cyc,n}}^{z_{2t,cyc,n}} E_t(\frac{1}{\rho_{cyc}})z_{2t,cyc,n} \cdot (a_1 z_{2t,cyc,n} + a_2)dz =$$
$$= \int_{z_{1t,cyc,n}}^{z_{2t,cyc,n}} (E_t(\frac{1}{\rho_{cyc}})a_1 z_{t,cyc,n}^2 + E_t(\frac{1}{\rho_{cyc}})a_2 z_{t,cyc,n})dz = E_t(\frac{1}{\rho_{cyc}})a_1 \frac{z_{2t,cyc,n}^3}{3} + E_t(\frac{1}{\rho_{cyc}})a_2 \frac{z_{2t,cyc,n}^2}{2} - \quad (19)$$
$$- E_t(\frac{1}{\rho_{cyc}})a_1 \frac{z_{1t,cyc,n}^3}{3} - E_t(\frac{1}{\rho_{cyc}})a_2 \frac{z_{1t,cyc,n}^2}{2}.$$

Substituting expressions $N_{1c,d,cyc,n}$ and $N_{2c,d,cyc,n}$ in $N_{c,d,cyc,n}$ we obtain the force perceived by the stretched zone of the element by oblique bending

$$N_{t,d,cyc,n} = E_t(\frac{1}{\rho_{cyc}})b\frac{z_{1t,cyc,n}^2}{2} + E_t(\frac{1}{\rho_{cyc}})a_1 \frac{z_{2t,cyc,n}^3}{3} + E_t(\frac{1}{\rho_{cyc}})a_2 \frac{z_{2t,cyc,n}^2}{2} -$$
$$- E_t(\frac{1}{\rho_{cyc}})a_1 \frac{z_{1t,cyc,n}^3}{3} - E_t(\frac{1}{\rho_{cyc}})a_2 \frac{z_{1t,cyc,n}^2}{2}. \quad (20)$$

The bending moment from the neutral line for the compressed zone in the normal calculated cross-section is equal to

$$M_{c,d,cyc,n} = M_{1c,d,cyc,n} + M_{2c,d,cyc,n}, \quad (21)$$

Where: $M_{1c,d,cyc,n}, M_{2c,d,cyc,n}$ – the moment of the compressed zone of the element of timber in different parts of the calculated cross-section with oblique bending on the n– th cycle of application of few cyclic reloads.

The cross-sectional area of the compressed zone of the element of timber in the form of a parallelogram will perceive the component of the bending moment

$$M_{1c,d,cyc,n} = \int_0^{z_{1c,cyc,n}} f_1(u_{c,d,cyc,n})z_{c,cyc,n}dA = \int_0^{z_{1c,cyc,n}} f_1(u_{c,d,cyc,n})bdz =$$
$$= b\int_0^{z_{1c,cyc,n}} \left(k_1(\frac{1}{\rho_{cyc}})z_{c,cyc,n}^2 + k_c(\frac{1}{\rho_{cyc}})^2 z_{c,cyc,n}^3 \right)dz = b(k_1(\frac{1}{\rho_{cyc}})\frac{z_{1c,cyc,n}^3}{3} + k_c(\frac{1}{\rho_{cyc}})^2 \frac{z_{1c,cyc,n}^4}{4}). \quad (22)$$

The cross-sectional area of the compressed zone of the element of timber in the form of a triangle will perceive the component of the bending moment

$$
\begin{aligned}
M_{2c,d,cyc,n} &= \int_{z_{1c,cyc,n}}^{z_{2c,cyc,n}} f_1(u_{c,d,cyc,n}) z_{c,cyc,n} dA = \int_{1c,cyc,n}^{z_{2c,cyc,n}} f_1(u_{c,d,cyc,n}) z_{c,cyc,n} \cdot f(b) dz = \\
&= b \int_{1c,cyc,n}^{z_{2c,cyc,n}} \left(k_1(\frac{1}{\rho_{cyc}}) z^2_{c,cyc,n} + k_c(\frac{1}{\rho_{cyc}})^2 z^3_{c,cyc,n} \right) \cdot (a_1 z_{c,cyc,n} + a_2) dz = \\
&= \int_{1c,cyc,n}^{2c,cyc,n} (k_1(\frac{1}{\rho_{cyc}}) a_1 z^3_{c,cyc,n} + k_1(\frac{1}{\rho_{cyc}}) a_2 z^2_{c,cyc,n} + k_2(\frac{1}{\rho_{cyc}})^2 a_1 z^4_{c,cyc,n} + k_2(\frac{1}{\rho_{cyc}})^2 a_2 z^3_{c,cyc,n}) dz = \\
&= k_1(\frac{1}{\rho_{cyc}}) a_1 \frac{z^4_{2c,cyc,n}}{4} + k_1(\frac{1}{\rho_{cyc}}) a_2 \frac{z^3_{2c,cyc,n}}{3} + k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^5_{2c,cyc,n}}{5} + k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^4_{2c,cyc,n}}{4} - \\
&- k_1(\frac{1}{\rho_{cyc}}) a_1 \frac{z^4_{1c,cyc,n}}{4} - k_1(\frac{1}{\rho_{cyc}}) a_2 \frac{z^3_{1c,cyc,n}}{3} - k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^5_{1c,cyc,n}}{5} - k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^4_{1c,cyc,n}}{4}.
\end{aligned}
\tag{23}
$$

Substituting formulas $M_{1c,d,cyc,n}$ and $M_{2c,d,cyc,n}$ in $M_{c,d,cyc,n}$ we obtain the moment that will perceive the compressed zone of the calculated cross section of the element of timber at an oblique bend

$$
\begin{aligned}
M_{c,d,cyc,n} &= b\left(k_1(\frac{1}{\rho_{cyc}}) \frac{z^3_{1c,cyc,n}}{3} + k_c(\frac{1}{\rho_{cyc}})^2 \frac{z^4_{1c,cyc,n}}{4} \right) + k_1(\frac{1}{\rho_{cyc}}) a_1 \frac{z^4_{2c,cyc,n}}{4} + k_1(\frac{1}{\rho_{cyc}}) a_2 \frac{z^3_{2c,cyc,n}}{3} + \\
&+ k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^5_{2c,cyc,n}}{5} + k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^4_{2c,cyc,n}}{4} - k_1(\frac{1}{\rho_{cyc}}) a_1 \frac{z^4_{1c,cyc,n}}{4} - k_1(\frac{1}{\rho_{cyc}}) a_2 \frac{z^3_{1c,cyc,n}}{3} - \\
&- k_c(\frac{1}{\rho_{cyc}})^2 a_1 \frac{z^5_{1c,cyc,n}}{5} - k_c(\frac{1}{\rho_{cyc}})^2 a_2 \frac{z^4_{1c,cyc,n}}{4}.
\end{aligned}
\tag{24}
$$

The total bending moment relative to the neutral line for the stretched zone in the normally calculated cross-section of the timber element is equal to

$$M_{t,d,cyc,n} = M_{1t,d,cyc,n} + M_{2t,d,cyc,n}, \tag{25}$$

Where: $M_{1c,d,cyc,n}, M_{2c,d,cyc,n}$ – components of the moment of the stretched zone of the element of timber in different parts of the calculated cross section with oblique bending on the n-th cycle of application of low-cycle reloads.

The cross-sectional area of the stretched zone of the element of timber in the form of a parallelogram will perceive the component of the bending moment

$$
\begin{aligned}
M_{1t,d,cyc,n} &= \int_0^{1t,cyc,n} f_2(u_{t,d,cyc,n}) dA = \int_0^{1t,cyc,n} f_2(u_{t,d,cuc,n}) b z_{t,cyc,n} dz = \\
&= \int_0^{1t,cyc,n} E_t(\frac{1}{\rho_{cyc}}) z^2_{1t,cyc,n} b dz = E_t(\frac{1}{\rho_{cyc}}) b \frac{z^3_{1t,cyc,n}}{3}.
\end{aligned}
\tag{26}
$$

The cross-sectional area of the stretched zone of the wooden element in the form of a triangle will perceive the component of the bending moment

$$M_{2t,d,cyc,n} = \int_{1t,cyc,n}^{2t,cyc,n} f_2(u_{t,d,cyc,n}) dA = \int_{1t,cyc,n}^{2t,cyc,n} f_2(u_{t,d,cuc,n}) z_{t,cyc,n} \cdot f(b) dz =$$
$$= \int_{1t,cyc,n}^{2t,cyc,n} (E_t(\frac{1}{\rho_{cyc}}) a_1 z_{t,cyc,n}^3 + E_t(\frac{1}{\rho_{cyc}}) a_2 z_{t,cyc,n}^2) dz = E_t(\frac{1}{\rho_{cyc}}) a_1 \frac{z_{2t,cyc,n}^4}{4} + \tag{27}$$
$$+ E_t(\frac{1}{\rho_{cyc}}) a_2 \frac{z_{2t,cyc,n}^3}{3} - E_t(\frac{1}{\rho_{cyc}}) a_1 \frac{z_{1t,cyc,n}^4}{4} - E_t(\frac{1}{\rho_{cyc}}) a_2 \frac{z_{1t,cyc,n}^3}{3}.$$

Substituting formulas $M_{1c,d,cyc,n}$ and $M_{2c,d,cyc,n}$ in $M_{t,d,cyc,n}$ we obtain the moment that the stretched area of the calculated cross-section of the element of timber with oblique bending will perceive

$$M_{t,d,cyc,n} = E_t(\frac{1}{\rho_{cyc}}) b \frac{z_{1t,cyc,n}^3}{3} + E_t(\frac{1}{\rho_{cyc}}) a_1 \frac{z_{2t,cyc,n}^4}{4} +$$
$$+ E_t(\frac{1}{\rho_{cyc}}) a_2 \frac{z_{2t,cyc,n}^3}{3} - E_t(\frac{1}{\rho_{cyc}}) a_1 \frac{z_{1t,cyc,n}^4}{4} - E_t(\frac{1}{\rho_{cyc}}) a_2 \frac{z_{1t,cyc,n}^3}{3}. \tag{28}$$

To determine the ultimate bending moment by formula $M_{t,d,cyc,n}$, which can be perceived by the beam, you need the value of the relative deformations at which the condition of equilibrium of forces $N_{c,d,cyc,n} = N_{t,d,cyc,n}$ substitute in the formula $M_{c,d,cyc,n}$, $M_{t,d,cyc,n}$.

CONCLUSIONS

Based on the analysis of the elements of timber, working under the action of repeated oblique transverse bending, the following conclusions can be drawn:

1. The current calculation norms of solid or glued timber elements that are in direct or oblique transverse bending under single or repeated loads do not take into account the actual operation of such elements, in particular the formation of folds in the compressed zone of pure bending, and give a significant margin of safety.
2. The accepted concept of "calculated cross-section" allows using the accepted law of distribution of deformations on the height of cross-section and layer-by-layer connection between deformations and stresses operating in a section of a bending element from timber taking into account the formation of folds in a compressed zone loads.
3. A method for calculating wooden beams from solid and glued wood using a deformation model, which takes into account the distribution of stresses in height in the compressed and stretched zones of the calculated cross-section and provides for the formation of folds in the compressed zone of the pure bending zone.
4. It is urgent to continue systematic studies of the complete physical diagram of the work of wood of different grades and strength class in compression and tensile strength to complete destruction under the action of static, low- and multi-cycle external loads; obtaining statistically reliable data to establish the functional relationship between stresses and strains under different temperature and humidity operating conditions.

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