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WOOD STRENGTH LIMITATION

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ABSTRACT

The limitation between independently measured wood strength tensor components is presented in the paper. The limitations can be observed among tensile and shear components in basic wood sections. All calculated limitations for nine hardwood species were lower than theoretically expected value 2π, except birch wood in radial section. Due to non-standard conditions during measurements of wood strength components, the deviations of calculated limitations for measured data from the expected theoretical value were observed.

Key words: wood strength, tension, shear, hardwood, physics of wood.

INTRODUCTION

The wood strength breaking is a process associated with loading force and loaded area. Before the wood specimen is loaded the loaded area is measured. The loading force is measured using testing machine. The static measurement of loading force is based on equilibrium between a force produced by machine and wood specimen. The process of area and force measurements is followed by evaluation of the wood strength. The wood strength is calculated using the definition formula. The process of wood strength evaluation is finished by comparing the calculated wood strength to the wood strength when the moisture content is 12% (POŽGAJ 1982). Wood is understood as a working substance in the wood strength evaluation process. The new surface of wood is formed spontaneously and work must be done to repair the wood specimen to its original state.

Perhaps, the wood strength is the most observable property of wood and its data is published for various kinds of wood species and loadings (KÚDELA 2010). Looking at such data, distortion in symmetry of wood strength as a second-order stress tensor can be found. Following the mentioned information the limitation of wood strength or its value data for various kinds of loading is investigated. The aim of this paper is to present the limitation between the wood strength tensor components. The null hypothesis states that there is no connection between wood strength tensor components in opposition to alternative hypothesis that there is at least one connection between the wood strength tensor components.

METHOD OF LIMITATION

The external force is in balance with the internal force of wood. The wood strength is not broken immediately at the beginning of measurements. The successive steps of stress are applied to wood until wood strength is broken. Stress is a linear transform of the force **f** to the surface. The surface is defined by its normal **n**. The magnitude of the normal is the area of its surface. The formula defining the transform of the force to the surface unit normal in xy plane is as follows:

$$\begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} n_x \\ n_y \end{pmatrix} \quad (1)$$

The first index of the stress tensor component is the force direction; the second index determines the normal direction.

The loading is called according to the arrangement of the surface normal and internal force. Tension σ is determined by parallel arrangement of normal directing out of the surface and internal force with the same direction as the direction of normal. Shear τ is described by perpendicular arrangement of the normal directing out of the surface and internal force perpendicular to it (Fig. 1).

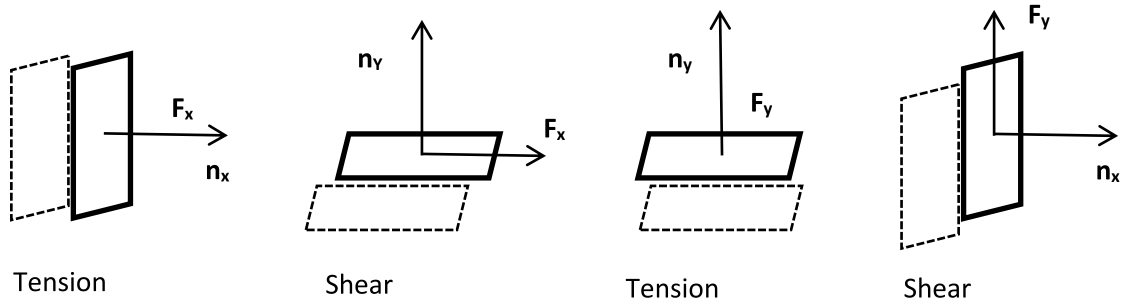

Fig. 1 The arrangement of loading force and normal of the surface

In general, the wood strength is probably most often broken in the processes of woodworking or wood machining. For example, tool moves inside the wood specimen around circular path during wood planning. The wood shavings are produced and removed from wood sample. At least one wood stress tensor component must be broken to produce wood shavings. On the other hand, in order to remove wood shavings, all components must be broken (Fig. 1). In general, the path of cutting edge is assumed to be closed in wood or in special case, it is circular (Fig. 2).

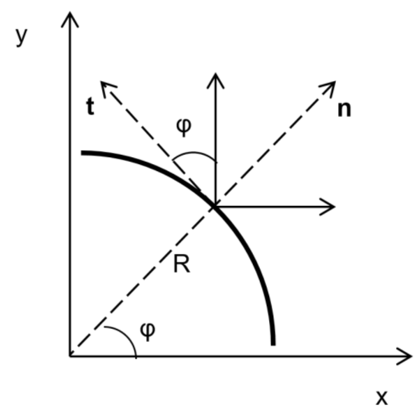

Fig. 2 Curve path of cutting edge in xy – coordinate system

Coordinates of the point on circular path with the origin [x_s; y_s] and R radius are [x; y]:

$$x = Rcos(\varphi) + x_s$$

$$y = Rsin(\varphi) + y_s$$

Tangent **t** of unit length to circular path in the point [x, y] is characterized by its components [-sin(φ); cos(φ)]:

$$dx = -Rsin(\varphi)d\varphi$$

$$dy = R cos(\varphi) d\varphi$$

because Rdφ is a differential of the path length ds:

$$ds = \sqrt{dx^2 + dy^2}$$

Normal **n** to the surface is perpendicular to tangent. The components of normal are [cos(φ); sin(φ)]. Finally, following expressions result from formula 1:

$$f_x = \sigma_{xx} cos(\varphi) + \sigma_{xy} sin(\varphi)$$

$$f_y = \sigma_{yx} cos(\varphi) + \sigma_{yy} sin(\varphi)$$

$$\vec{f} = f_x \vec{\imath} + f_y \vec{\jmath}$$

Therefore, applied force is projected into direction of normal and direction of tangent. The magnitude of normal force $\mathbf{f_n}$ is a dot product of applied force and normal:

$$|\vec{f_n}| = \vec{f} \cdot \vec{n} = \sigma_{xx} cos^2(\varphi) + (\sigma_{xy} + \sigma_{xy}) sin(\varphi) cos(\varphi) + \sigma_{xx} sin^2(\varphi)$$

The formula for normal force is:

$$\vec{f_n} = |\vec{f_n}| \cos(\varphi) \vec{\imath} + |\vec{f_n}| \sin(\varphi) \vec{\jmath}$$

The magnitude tangent force $\mathbf{f_t}$ is a dot product of applied force and tangent.

$$|\vec{f_t}| = \vec{f} \cdot \vec{t} = \sigma_{yx} cos^2(\varphi) + (\sigma_{yy} - \sigma_{xx}) sin(\varphi) cos(\varphi) - \sigma_{xy} sin^2(\varphi)$$

The formula for tangent force is:

$$\vec{f_t} = -|\vec{f_t}| \sin(\varphi) \vec{\imath} + |\vec{f_t}| \cos(\varphi) \vec{\jmath}$$

Without loss of generality, for the purpose of the research wood on average is assumed as a homogeneous material. Otherwise wood strength components must be defined as a function of coordinates. Constant components of wood strength tensor are the goal of the following derivation due to wood property. The magnitude of applied tangent force on definite interval of path $\langle 0, \varphi \rangle$ is expressed as integral of infinitesimal tangent forces along this path:

$$|\vec{F_t}| = T \int_0^{\varphi} \boldsymbol{f_t} \cdot \boldsymbol{ds} = TR \int_0^{\varphi} |\vec{f_t}| d\varphi$$

The symbol T is used to describe wood dimension perpendicular to xy plane. The work W of tangent force is expressed as contour integral along the definite interval of path $\langle 0, \varphi \rangle$.

$$W(\varphi) = TR^2 \int_0^{\varphi} \int_0^{\varphi} |\vec{f_t}| d\varphi \, d\varphi$$

Finally, the work as a function of angle is expressed:

$$W(\varphi) = TR^2 \left(\left(\frac{\sigma_{xx} - \sigma_{yy}}{8} \right) sin(2\varphi) - \left(\frac{\sigma_{xx} - \sigma_{yy}}{4} \right) \varphi + \left(\frac{\sigma_{yx} - \sigma_{xy}}{4} \right) \varphi^2 - \left(\frac{\sigma_{yx} + \sigma_{xy}}{8} \right) (cos(2\varphi) - 1) \right)$$

The only work is done by tangent force, because normal force is always perpendicular to the direction of path. Moreover, the work is zero along any closed path, because the wood strength is a property and applied force is conservative. The form of wood strength limitation is:

$$2\pi = \frac{\sigma_{xx} - \sigma_{yy}}{\sigma_{yx} - \sigma_{xy}} \quad (2)$$

Three different limitations are useful for wood as an orthotropic material:

$$2\pi = \frac{\sigma_{LL} - \sigma_{RR}}{\sigma_{RL} - \sigma_{LR}}$$
$$2\pi = \frac{\sigma_{RR} - \sigma_{TT}}{\sigma_{TR} - \sigma_{RT}} \quad (3)$$
$$2\pi = \frac{\sigma_{TT} - \sigma_{LL}}{\sigma_{LT} - \sigma_{TL}}$$

Afterwards, limitation for shear strength components must be according to this formula:

$$0 = (\sigma_{RL} - \sigma_{LR}) + (\sigma_{TR} - \sigma_{RT}) + (\sigma_{LT} - \sigma_{TL}) \quad (4)$$

Longitudinal, radial, tangential directions are designated by subscripts L, R, T.

EXPERIMENTAL VALIDATION OF LIMITATION

The exactness of derived limitation should be proved by experiments. The experimental data for validation were mentioned in publication of KÚDELA (2010). Sufficient list of measured wood strength in tension and shear loadings are given in the publication. The tested specimens represent the most utilized ring-porous and diffuse-porous tree species in Slovakia (except black locust, alder and willow). The right side of limitations as they were calculated following experimental data of different wood species are shown in Table 1.

Tab. 1 Computed right hand side of limitation from experimental data for different hardwood species.

Species	Wood strength components of right hand side of limitation	Computed right hand side of limitation	Desired left hand side of limitation
Beech	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	5.331	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	–15.49	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.599	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–0.11	0
Hornbeam	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	5.868	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	27.2	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.72	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–1.32	0
Maple	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	6.228	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	–11.3	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.09	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–5.7	0
Birch	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	6.656	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	8.9	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	6.24	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–1.69	0
Limetree	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	5.234	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	25.2	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	4.65	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–3.14	0
Poplar	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	5.568	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	Division by zero	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.54	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–0.31	0

Oak	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	5.77	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	5.6	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.8	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–0.55	0
Ash	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	6.16	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	1.0	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5,7	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–1.36	0
Elm	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	6.04	2π
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	51.3	
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	5.4	
	σ_{RL} σ_{LR} σ_{TR} σ_{RT} σ_{LT} σ_{TL}	–2.67	0

The right side of limitation from experimental data of poplar in cross section cannot be calculated because of equal values of shear strengths in perpendicular to grains of radial and tangential sections (4.10 MPa) and different values of tensile strengths perpendicular to grains.

THE ANGLE OF STRENGTH VECTOR ACTION

The matrix on the left side of formula 1 is called as strength vector if and only if the right side of formula 1 is a result of multiplication of wood strength matrix and matrix of normal in which strength vector is applied. In general, all forces projected to the normal and equal to strength vector, break the wood strength. But only one force is equal to strength vector and its position is unique. The mentioned fact is illustrated in Fig. 3.

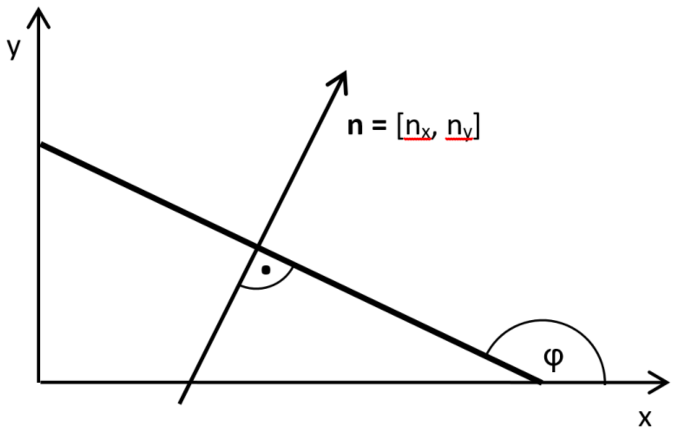

Fig. 3 The position of loaded surface and reference frame, angle φ of strength vector is defined by the components of the normal n of the surface affected by strength

The angle φ of strength vector is according to the formula 5:

$$\sigma_{yx} tg^2(\varphi) + (\sigma_{xx} - \sigma_{yy}) tg(\varphi) - \sigma_{xy} = 0 \quad (5)$$

The root of formula 5 must be according to Fig. 3. Therefore, the angle of strength vector is:

$$\varphi = \arctan\left(\frac{-(\sigma_{xx}-\sigma_{yy})-\sqrt{(\sigma_{xx}-\sigma_{yy})^2+4\sigma_{xy}\sigma_{yx}}}{2\sigma_{yx}}\right)$$

The angle values of strength vector are shown in Table 2.

Tab. 2 The angle of strength vector action of various hardwood species according to experimental data.

Species	σ_{xx} σ_{yy} σ_{yx} σ_{xy}	Angle of strength vector action in degrees
Beech	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	106.2
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	124.6
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	104.6
Hornbeam	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	106.0
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	128.4
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	104.8
Maple	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	105.7
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	124.4
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	105.9
Birch	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	103.2
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	130.6
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	102.7
Limetree	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	105.4
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	125.7
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	105.7
Poplar	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	104.4
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	131.1
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	103.5
Oak	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	104.2
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	132.0
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	103.4
Ash	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	103.2
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	128.5
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	102.6
Elm	σ_{LL} σ_{RR} σ_{RL} σ_{LR}	103.5
	σ_{RR} σ_{TT} σ_{TR} σ_{RT}	134.6
	σ_{TT} σ_{LL} σ_{LT} σ_{TL}	102.8

All strength vector angle values are greater than right angle.

DISCUSSION

Physical treatment of wood strength provides the interesting results. The conclusions are derived without regard to wood strength dependency on wood microscopic structure (SALMÉN and BURGERT 2009), chemical composition (WINANDY and ROWELL 2005), oven dry density (ZHANG 1997) or anatomical direction (KIM 1986). The published data of KÚDELA (2010) provided the comprehensive set of information about wood strength for selected hardwoods.

The independency of tensile strength in different anatomical directions results from different movement of possible wooden parts after breaking the strength. The same conclusion can be stated for independency of shear strength components.

The wood strength components are results of measurement and they are random variables. Error propagation can happen easily in the process of calculation using measured values. Therefore, only averages were used in calculating. The particular coincidence is between theory and experiment in longitudinal sections. Value 2π was only broken in the case of birch in radial plane. All other recalculated experimental results underestimated 2π value. The worse results were gathered in cross section. Only measurement results gathered for oak wood in cross section were close to 2π value. It is difficult to say where the errors occurred during limitation. Direction of normal to the surface and applied force direction do not coincide can be considered one of the reasons. The direction of strength vector does not coincide with the applied force. In fact, wood is cylindrical orthogonal anisotropic material and the previous errors can be eliminated easier in longitudinal sections than in cross section. The derived limitations are valid in a point and its local area. The limitation for shear strength components is also not completely equal zero. The experimental data provided only negative values of right side of limitation. The closest values of zero are estimated in shear strength values of beech wood.

The issue of validity of limitation for isotropic materials used in wood processing industry can be discussed. Such materials are anisotropic (plywood, particleboard, fibreboard, etc) in plane or fully isotropic (metal pieces). The limit of two zeros in nominator and denominator of fraction (2) is also equal to 2π can be suggested.

Wood strength in compression perpendicular to grain is difficult to measure because of three phase strain – stress diagram of wood especially in radial direction. The limitation of wood strength must be also useful for loading in compression instead of tension. It seems more suitable to define the ultimate limit of wood in compression perpendicular to grain in natural way according to wood strength limitation. However, such attitude requires suitable experimental method for wood testing in compression. The experimental method is beyond the scope of this article.

CONCLUSION

All wood strength components are measured independently with prescribed method. Tensile and shear wood strength tensor components show mutual constraint. Such wood strength limitation was observed in the best conformity in longitudinal sections. Almost all observed hardwood strength delivered lower value than 2π in calculated limitation for experimental data, except the result of limitation for birch strengths in radial section. The calculated limitation for shear strength was always negative for all observed species. The deviations of experimental and theoretical results are due to imperfect arrangement of loading force and strength vector during testing of individual wood strength components.

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