APPROACH TO DESIGNING AN UPHOLSTERED FURNITURE FRAME BY THE FINITE ELEMENT METHOD

Nelly Staneva - Yancho Genchev - Desislava Hristodorova

ABSTRACT

An approach to designing an upholstered furniture frame using the finite element method is presented in the paper. Two 3D discrete models of one-seat upholstered furniture frame were created without and with strengthening details under the upper rails of the seat. Linear static analyses were carried out with CAE system Autodesk Simulation Mechanical® by the finite element method (FEM) simulating light-service loading. The orthotropic characteristics of pine solid wood (*Pinus Sylvestris* L.) for the rails and particle boards (PB) for the side plates are provided in the analyses. FEA was performed with regard to laboratory established coefficients of rotational stiffness of used corner joints - butt joints and end to face butt joints with staples and PVA glue. As a result, the distribution of stresses, displacements and equivalent strains for both discrete models of upholstered furniture frame with staple corner joints is presented and analysed. The deformation and strength behaviour of the upholstered furniture frame is improved by the strengthening of the upper rails of the seat. The created new approach to designing upholstered furniture frames with taking into account the pliability of staple corner joints by FEM gives correct results and can be successfully used in optimisation of upholstered furniture constructions.

**Key words:** upholstered furniture frame, staple corner joints, stress, strain, FEM.

INTRODUCTION

Upholstered frame has to be both strong and stiff enough to bear the static and dynamic loads in service. Strength and deformation characteristics of upholstered frame are very important to ensure optimal design of upholstered furniture.

In our days strength design of furniture constructions can be accomplished by 3D solid modeling and structural analysis software, based on finite element method (FEM). Eckelmann and Suddarth are the first to propose the use of numerical methods for furniture construction, preparing a special package of programs in Fortran IV—CODOFF and CODOC-3 (*SMARDEWSKI* 1998).

First attempts of numerical investigation of construction frame of upholstered furniture were undertaken by *SMARDEWSKI* (2001): he has carried out finite element analysis of the skeleton of 1-seat armchair that resulted in lower material consumption and assures optimal strength of the construction using Algor® CAE program, but he has modeled the materials of the construction elements (birch wood and chipboards) as isotropic brick finite elements, that is unjustified. The discrete sofa model do not consider the real behavior of the corner joints of the sofa construction elements. Further he has used Algor® CAE for optimization
of dimensions of the main construction elements of 2-seat sofa frame modeled with brick orthotropic finite elements (HDF boards for seat and backrest, particle boards for side elements and pine and beech wood for beam elements) and has concentrated the finite elements network in places where elements of different thicknesses and cross sections are connected to achieve an accurate picture of the state of strains and stresses (SMARDZEWSKI, PREKRAT 2009).

KASAL (2006) has investigated the strength properties of glued-dowel joined sofa frames constructed of solid wood and wood based composite materials by using RISA 3D finite element analysis software. Considering wood materials as isotropic, he has established that the OSB (18 mm thick) has lowest load bearing capacity. The failure of OSB sofa frame has been the pull-out of dowels from the member with some core wood particles attached to the dowel and some splits have occurred at the edge of the butt members in the sofa frames.

WANG (2007) has carried out nonlinear static analysis using finite element software SAP2000 of three configurations of 3-seat sofa skeleton made entirely of OSB under three different loads. The sofa frame is modeled by 3D beam elements with two types of connections (screws with metal-plates and staples with metal-plates). Wang has used rigid and semi-rigid types of links in the models, but she has introduced in the program the experimentally determined linear-elastic stiffness of joints rather than the rotational.

YILDIRIM et al. (2015) investigated the fatigue behaviour of M&T joined armchair frames constructed of Scots Pine with isotropic characteristics by ANSYS Workbench software. The results show that there are close convergence between experimental study and FEM results - the consistency level was 81.25%.

In Bulgaria first attempt to study wooden chair frames by FEM was made by MARINOVA, KUJTSCHUKOV (1999) using the developed by MARINOVA (1997) methodology of stress and strain furniture structure analysis of case furniture on the base of FEM. The methodology is adapted to specific characteristics of furniture structure taken into account the pliability of furniture corner joints under loading – test established spring constants of elastic fixation have been imported in the computer program SAP 90.

The aim of the presented study was to define an approach to designing a frame of upholstered furniture taking into consideration the experimentally established coefficients of rotational stiffness of used staple joints by CAE system Autodesk Simulation Mechanical® based on the Method of Finite Elements (FEM).

**MATERIAL AND METHODS**

**Theoretical model**

The current CAE software programs including Autodesk Simulation Mechanical® predominantly implement the method of finite elements (FEM) through the principle of full potential energy (variance principle) (MARKOV 1998) - from all possible configurations of a conservative mechanical system, equilibrium is one in which we have the stationary full potential energy of the system, i.e. small changes in displacements do not cause a change in full potential energy. The condition for the minimization of full potential energy is presented as the following equation system:

\[
[F] = [C].[D],
\]

where \( [F] \) is the vector of applied loads and boundary conditions;

\( [D] \) - vector of nodal linear displacements and rotations;

\( [C] \) - global stiffness matrix.

The applied loads are determined by:
\[ \{F\} = \{F_n\} + A^m_{e=1} \{f^e\}, \]  

(2)

where \(\{F_n\}\) is the vector of applied nodal forces and moments;  
\(\{f^e\}\) - element level applied force vector;  
\(A^m_{e=1}\) - assembly operator;  
\(m\) - number of finite elements.

The global stiffness matrix of the construction is obtained superposing by nodes of the components of the stiffness matrices of all elements by the assembling operator:

\[ [C] = A^m_{e=1} \{c^e\}, \]  

(3)

where \(\{c^e\}\) is the element level stiffness matrix.

The matrix equation (1) is solved for the vector of displacements from the vector of applied nodal forces and the global stiffness matrix of the structure. A major computational problem is the receiving of components of stiffness matrices by elements. For every finite element the following equation system is in effect:

\[ \{f^e\} = [c^e].\{d^e\} \]  

(4)

In the case of a thin orthotropic plate of thickness \(t\) with dimensions \(a\) (axis \(x\)) and \(b\) (axis \(y\), 12 DOF’s, for which the construction elements (rails and side plates) of the skeleton of upholstered furniture are adopted, as accepted in the methodology of Marinova (1996) for structural elements of the case furniture, displacements at any point of the plate element are presented by approximating functions - polynomials with 12 unknown coefficients. The nodal displacements are determined from the equation:

\[ \{d^e\} = [A].\{\alpha\}, \]  

(5)

where \([A]\) is the matrix of the form function;  
\(\{\alpha\}\) - vector of unknown coefficients in the approximating functions.

The relationship between deformations and displacements at any point of the plate element, expressed by the nodal displacements is presented with the known from the theory of elasticity dependency:

\[ \{\varepsilon(x, y)\} = [B].\{d^e\}, \]  

(6)

where \([B]\) is the matrix, derived from the differentiation of the matrix \([A]\);  
\(\{\varepsilon(x, y)\}\) - relative linear and angular deformations.

The relationship between stresses and deformations at any point of the plate element, expressed by the nodal displacements is:

\[ \{\sigma(x, y)\} = [K]\{\varepsilon(x, y)\} = [K].[B].\{d^e\}, \]  

(7)

where \([K]\) is the stiffness matrix.

The stiffness matrix \([K]\) for an orthotropic plate finite element has the appearance:

\[ [K] = \begin{bmatrix} K_x & K & 0 \\ K & K_y & 0 \\ 0 & 0 & K_{xy} \end{bmatrix}. \]  

(8)

where: \(K_x = \frac{E_t t^3}{12(1-\nu_y \nu_y)}\) and \(K_y = \frac{E_t t^3}{12(1-\nu_x \nu_y)}\) are the cylindrical stiffness of the plate in the main directions of orthotropy - \(x\) and \(y\):  
\(K_1 = K_y \nu_y = K_y \nu_x\) - stiffness, accounting for the effect of Poisson;  
\(K_{xy} = \frac{1}{2} (1 - \sqrt{\nu_x \nu_y}) \sqrt{K_x K_y}\) - torsion stiffness of the plate.

The relation between the engineering elastic constants for the wood based plates is (Bodig, Jayne 1982):

\[ \nu_x, E_y = \nu_y, E_x. \]  

(9)
where \( v_x \) is the Poisson’s ratio in direction \( x \);
\( v_y \) - Poisson’s ratio in direction \( y \);
\( E_x \) - modulus of relative linear deformation in the main direction of orthotropy \( x \);
\( E_y \) - modulus of relative linear deformation in the main direction of orthotropy \( y \).

The stiffness matrix of the plate element is derived from eq.4 on the base of the energetic principle:

\[
[c^e] = \left[\int_0^a \int_0^b [B]^T \cdot [K] \cdot [B] \cdot dxdy\right],
\]

where \([B]^T\) is the transposed matrix of the matrix \([B]\).

**Discrete model**

3D discrete model of one-seat upholstered furniture frame (skeleton) was created with Autodesk Inventor Pro® and marked \textit{model A} – Fig. 1a. Used rails are with cross section 25×50 mm and the side plates are with thickness of 16 mm. Additionally to the \textit{model A} strengthening details under the upper rails of the seat with a shape of triangle prism were modelled and this discrete model was marked \textit{model B} – Fig. 1b. The generated Midplane meshes with plate finite elements have 5130 orthotropic finite elements and 33616 DOF's for \textit{model A} and 5230 orthotropic finite elements and 34096 DOF's for \textit{model B}.

A linear static analysis of both 3D discrete models of the upholstered furniture seat was carried out with CAE system Autodesk Simulation Mechanical® by the Finite Elements Method (FEM) according to described above theoretical model. Two design scenarios were performed.

Orthotropic materials type was used for construction elements of the skeleton for both models:

Scots pine (\textit{Pinus sylvestris L.}) for rails and strengthening details with measured density 435.50 kg/m³ according to BDS EN 323:2001 and elastic characteristics:

\( E_L = 9000.10^6 \text{ N/m}^2, \quad E_T = 593.10^6 \text{ N/m}^2, \quad G_{LT} = 554.5.10^6 \text{ N/m}^2, \quad v_{LR} = 0.03, \quad v_{LT} = 0.027, \quad v_{TL} = 0.41, \quad v_{RL} = 0.049. \)

Particleboard (PB) for side plates with thickness 16 mm and measured density 678.06 kg/m³ according to BDS EN 323:2001. The physical and mechanical characteristics of the used PB panels are: modulus of elasticity in bending \( E_x = 2700.10^6 \text{ N/m}^2 \) and \( E_y = 1600.10^6 \text{ N/m}^2 \); bending strength 11.10⁶ N/m²; Poisson ratios \( v_x = 0.30 \) and \( v_y = 0.18 \).

Support boundary conditions were set: bottom front rail – no translation on \( y \) direction and bottom rear rail no translation on \( x \), \( y \) and \( z \) direction – Fig.1.

In order to simulate semi-rigid connections between rails and side plates of the skeleton two actions were performed:

First – in the place of joints in the discrete model narrow zones were created with established via tests by FEM lower modules of elasticity of the used materials perpendicular to the common edge of the corner joint.

Second - the laboratory determined by Hristodorova (2018) coefficients of rotational stiffness of the corner joints with 2 staples and PVA glue, loading under compression were introduced in the nodes of the respective corner joints - case butt joints \( (c = 1017.52 \text{ N.m/rad}) \) and end to face butt joints \( (c = 827.77 \text{ N.m/rad}) \). The coefficients of rotational stiffness of the corner joints in the compression bending were laboratory established and calculated according the method described in Jivkov, Marinova (2006).

The both discrete skeleton models were loaded with a total load of 800 N, distributed as follows (Fig. 1a, b):

\textit{Seat:} 80% was set as a remote force, distributed between upper rails of the seat with application point of 100 mm in front of the upper rear rail, simulating upholstery base made of zig-zag springs;
**Backrest:** 16% set as equal nodal forces, distributed on the edges of the two sides of the backrest, simulating elastic belts.

![Fig. 1 FEM models A and B of the upholstered furniture frame and loading.](image)

For validation of the proposed approach through comparing the vertical displacements with the laboratory ones a third design scenario was carried out loading discrete *model B* with a total load of 800 N distributed only on the upper rails of the seat with application point of 100 mm in front of the upper rear rail – Fig. 1c.

### RESULTS AND DISCUSSION

The results of static analysis for linear displacements $u$, nodal rotations $\theta$, maximum principal stresses $\sigma_1$, minimum principal stresses $\sigma_2$ and equivalent strains $\varepsilon$ for *model A* and *model B* are shown in Tab. 1 and Tab. 2, Fig. 2 to Fig. 7. The visualizations of the deformed model are shown with a scale factor 3% of model size and 5% for side plates.

<table>
<thead>
<tr>
<th>Parameters and location</th>
<th>$u_x$, [mm] side plates</th>
<th>$u_x$, [mm] front upper rail rear upper rail</th>
<th>$u_z$, [mm] base of side plates</th>
<th>$\theta_x$, [°] rear upper rail side plates</th>
<th>$\theta_y$, [°] front upper rail rear upper rail</th>
<th>$\varepsilon$, [m/m] side plates</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model A</strong></td>
<td>0.10</td>
<td>−1.34</td>
<td>−2.68</td>
<td>0.41</td>
<td>0.79</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Model B</strong></td>
<td>0.13</td>
<td>−0.96</td>
<td>−2.01</td>
<td>0.77</td>
<td>0.56</td>
<td>0.25</td>
</tr>
</tbody>
</table>

In Fig. 2a, b the distribution of resultant linear displacement for both models is presented. The maximal resultant displacement for both models was received in the middle of the rear upper rail of the seat and is determined mainly by the y-displacement ($u_y$) – Tab. 1. The strengthening of the upper rails of the seat leads to reducing of the resultant displacements of front and rear rails of the seat 1.4 times. In the side plates the maximal resultant linear displacements in the field of upper front and rear rails decrease 1.4 times for *model B*. In the base of the seat of side plates however 1.88 times greater $u_z$ displacements (absolute value of 0.36 mm) and dissolution of the side plates were established due to the redistribution of the load – Fig. 2a, b and Fig. 3.
Tab. 2 Maximal displacements and stresses in the side plates.

<table>
<thead>
<tr>
<th>Parameters and location</th>
<th>Model A</th>
<th>Model B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(u_x, \text{[mm]}) backrest</td>
<td>0.104</td>
<td>0.126</td>
</tr>
<tr>
<td>(u_y, \text{[mm]}) front upper rail</td>
<td>-0.13</td>
<td>-0.08</td>
</tr>
<tr>
<td>(u_y) backrest</td>
<td>-0.16</td>
<td>-0.11</td>
</tr>
<tr>
<td>(u_z, \text{[mm]}) base of side plates</td>
<td>0.41</td>
<td>0.77</td>
</tr>
<tr>
<td>(\theta_{res}, \text{[°]}) front upper rail</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>(\sigma_1, \text{[N/mm}^2) front upper rail</td>
<td>3.05</td>
<td>2.22</td>
</tr>
<tr>
<td>(\sigma_3, \text{[N/mm}^2) front upper rail</td>
<td>-4.12</td>
<td>-1.41</td>
</tr>
<tr>
<td>(\sigma_3, \text{[N/mm}^2) rear upper rail</td>
<td>-2.21</td>
<td>1.96</td>
</tr>
<tr>
<td>(\sigma_3, \text{[N/mm}^2) rear upper rail</td>
<td>-6.23</td>
<td>-2.53</td>
</tr>
</tbody>
</table>

Fig. 2 Linear resultant displacements for model A, model B and model B loaded only on the seat.

Fig. 3. Distribution of linear resultant displacements of side plates for model A and model B.

The established by FEM maximal linear vertical displacement of \(u_y^{FEM} = -2.52 \text{ mm}\) for the upper rear rail and \(u_y^{FEM} = -1.2 \text{ mm}\) for upper front rail (Fig. 2c) at load \(F = 800 \text{ N}\) distributed only on the upper rails of the seat with application point of 100 mm in front of the upper rear rail (Fig. 2c) were compared with corresponding laboratory received \(u_y^{lab} = -2.4 \text{ mm}\) and \(u_y^{lab} = -1.16 \text{ mm}\). For maximal linear z-displacement in the base of side plates the following results were received: \(u_z^{FEM} = -0.96 \text{ mm}\) and \(u_z^{lab} = -0.92 \text{ mm}\). It is
evident that the values of the numerical and laboratory displacements are very close and the differences are smaller than 5%.

The maximal values of the nodal rotations $\theta$ are given in Tab. 1. They are determined mainly by rotations around $z$-axis. The maximal resultant nodal rotations are located in the upper rear rail for both models - Fig. 4. The strengthening of the upper rails of the seat leads to decreasing of the nodal rotations of front and rear rails of the seat 1.3 times.

![Fig. 4 Distribution of rotation resultant displacements for model A and model B.](image)

In model A the maximal value of maximum principal stress $\sigma_1$ (tension) of 7.79 N/mm$^2$ and minimum principal stress $\sigma_3$ (compression) of 8.01 N/mm$^2$ are located in the middle of the upper rear rail of the seat, on the bottom and on the top, respectively - Fig. 5 and Fig. 6.

In model B the maximal values of maximum principal stress $\sigma_1$ of 7.18 N/mm$^2$ and minimum principal stress $\sigma_3$ of 8.29 N/mm$^2$ are located in the strengthening details of the upper rear rail of the seat. In the middle of the upper rear rail in the model B maximum principal stress and minimum principal stress are 1.2 times less than the same for model A – Fig. 5 and Fig. 6.

![Fig. 5 Distribution of maximum principal stresses for model A and model B.](image)

In the side plates maximal values of compression stresses are located in the field of upper rear rail of the seat for both models, as for model B they decrease 2.5 times – Tab. 2 and Fig.7. Maximal values of tension stresses in the side plates are located in the field of upper front rail of the seat for both models, as for model $B$ they decrease 1.4 times –Tab. 2.

The maximal equivalent strain $\varepsilon$ is located in the front of the side plates in the field of upper front rail for both model but for model B is 2.2 times less that the same in the model A – Table 1.
CONCLUSIONS

In the result of this study an approach is established and proposed to design of one-seat upholstered furniture frame made of Scots pine and PB taking into account laboratory established coefficients of rotational stiffness of used corner joints with staples and PVA glue by FEM with CAE program Autodesk Simulation Mechanical® and the following conclusions can be derived:

Under light-service load 80% located in the 100 mm in front of rear rail of the seat and 16% on the backrest the most loading construction part of the upholstered furniture frame is the rear upper rail of the seat where the maximum values for linear displacements, nodal rotations and stresses are received due to the nature of the applied force.

The strengthening of the upper rails of the seat improves the deformation and strength behaviour of the upholstered furniture frame – linear displacements in the upper rails of the seat decrease about 1.4 times, nodal rotations – 1.3 times, principal stresses – 1.2 times. In the field of upper rails of the side plates the displacements decrease about 1.4 times, principal stresses – 2.5 times.

The created new approach for designing of upholstered furniture frame with taking account of pliability of staple corner joints with CAE system Autodesk Simulation Mechanical® by the Finite Element Method (FEM) gives correct results and can be successfully used in optimisation of upholstered furniture constructions. This approach is a novelty in deformation and strength sizing of the frames of upholstered furniture.
REFERENCES


ACKNOWLEDGEMENT

This document was supported by the grant No BG05M2OP001-2.009-0034-C01, financed by the Science and Education for Smart Growth Operational Program (2014–2020) and co-financed by the EU through the ESIF.

AUTHORS’ ADRESS

Nelly Staneva, PhD.
Yanko Genchev, PhD.
Desislava Hristodorova
University of Forestry
Faculty of Forest Industry
Department of furniture production
1797 Sofia
Bulgaria
nelly_staneva@yahoo.com