STIFFNESS OF SOLID WOOD BEAMS UNDER DIRECT AND OBLIQUE BENDING CONDITIONS


ABSTRACT

The article is devoted to studying the features of the work of solid pine wood beams under direct and oblique bending. The study of oblique bending is represented by different angles of inclination. The destructive load of the beams under such conditions and the deformation parameters of bending elements were determined. Experimental results were compared with theoretical ones obtained as a result of calculations in the "LIRA" software complex. Study and analysis of calculations for different types of stiffnesses (with orthotropy, without orthotropy, and with the complete wood deformation diagram) using the software package was carried out. The optimal type of stiffnesses used in the "LIRA" software complex was determined based on the graphs for beam deformation. The results of experimental and theoretical studies can be used to design roof structure elements.

Keywords: solid wood; bearing capacity; oblique bending; bending stiffness.

INTRODUCTION

Wood is a renewable natural material, the use of which in various industries is growing significantly (Rudavska et al., 2018, Yasniy et al., 2022, Pinchevska et al., 2019, Homon et al., 2022, Homon et al., 2023, Wdowiak-Postulak 2020). Wood is significantly lighter compared to other materials such as concrete (Dvorkin et al., 2021), metal, various composites (Iasnii et al., 2023, Imbirovych et al., 2023). On the other hand, it is not inferior to them in terms of physical and mechanical properties. However, it prevails in many cases (De La Rosa Garsia et al., 2013; Sobczak-Piastka et al., 2020; Gomon et al., 2020). Bearing elements and structures made of wood are used in the construction of civil and industrial buildings (Vahedian et al., 2019, Soriano et al., 2016, Kudela 2017; Donadon et al., 2020), engineering structures (Bosak et al., 2022). Usually, coniferous species are used (pine, spruce, larch and others) (Homon et al., 2023, Janiak et al., 2023). Wooden elements and structures can work under various types of stress-deformed states (Sobczak-Piastka et al., 2023).

The bending of wooden elements and structures is one of the most common type of stress-deformed state in building practice (Betts et al., 2010, Gomon et al., 2022, Kúdela 2017). Such elements as floor beams, rafters, purlins, trusses work for direct (plane) and oblique bending. The study of such stress-deformed states is important today due to the increased use of wood in buildings. The direct bending of wooden beams has been studied
in (Gomon et al., 2019, Vahedian et al., 2019, Sobczak-Piastka et al., 2020). The work of bending elements during oblique bending needs to be studied more (experimental studies were not conducted). There are no complex studies of beams under oblique bending in the “Lira” software. This type of stress-strain state is complex and requires special attention of scientists (Kulman et al., 2017, Gomon et al., 2022, Pencik 2015, Gomon et al., 2023). And, therefore, additional experimental and theoretical studies.

The aim of the paper is to investigate the stiffness of solid wood beams made under the conditions of oblique and direct bending by experimental study and compare obtained results with results of mathematical calculations in the "LIRA SAPR" software complex. This program allows you to simulate the stress-strain state of various structures and elements.

**MATERIALS AND METHODS**

The experimental part of the research consisted in testing the solid wood beams for direct and oblique bending. An experimental installation for experimental studies was developed by Gomon et al., (2019).

Beams are made of C30 class pine with a length of 1650 mm (DBN B.2.6-161: 2017; NDS: 2018, EUROCODE 5:2004). The cross-section of the beams during oblique and direct bending tests was 50 x 80 mm. The marking of the experimental samples and the characteristics of their work are shown in Table 1.

<table>
<thead>
<tr>
<th>Characteristics of work</th>
<th>Beam name</th>
<th>Cross-sectional dimensions b х h (mm)</th>
<th>Angle of inclination</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct bend</td>
<td>B-1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B-2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>B-3</td>
<td>50x80</td>
<td>10°</td>
</tr>
<tr>
<td>Oblique bend</td>
<td>B-4</td>
<td>50x80</td>
<td>10°</td>
</tr>
<tr>
<td></td>
<td>B-5</td>
<td>50x80</td>
<td>25°</td>
</tr>
<tr>
<td></td>
<td>B-6</td>
<td>50x80</td>
<td>25°</td>
</tr>
</tbody>
</table>

The calculation scheme of the test in direct and oblique bending includes a freely lying beam on two supports (hinged movable and hinged fixed). Calculated beam span was 1500 mm. The load was applied in the middle third of the span by two concentrated forces. It allowed ensuring the operation excluding orthotropy of the beam under conditions of pure bending. The calculation scheme of the beam test is shown in Fig. 1.

**Fig.1** The calculation scheme of beams B-1…B-6.
The scheme of the experimental installation during direct bending tests is shown in Fig. 2.

![Scheme of the experimental installation for testing beams B-1, B-2 under direct bending.](image)

Hydraulic jack DOSM-5 was used to apply the load to the beams. The value of load to beams was monitored by used a dynamometer. The distribution of the load was carried out through a metal traverse. All equipment have passed state verification before used in experimental studies. The load was applied by steps of 8-10% of the estimated destructive load, taking into account regulatory documents (DBN B.2.6-161: 2017, NDS: 2018, EUROCODE 5:2004). All deflection gauges were measured after applying each degree of load. Deflection gauges were arranged in the middle of the span during direct bending tests. ICH-10M, 6PAO watch-type indicators were used to record the beam movement. Exposure was carried out for at least 5 minutes at each level of load.

Beams B-3...B-6 were tested under oblique bending for different angles of inclination. The angle of inclination was ensured by metal clips. Wooden supports were arranged between the metal brackets and the beam to prevent local crumpling of the wood.

The deflection gauges were arranged in the middle of the span in the places of loads applied and on the supports that allowed us to record deformations of the beam. The installation of deflection gauges was carried out in the plane and out of the plane of the beam to record movement in these planes. Ties were installed on the experimental beams at the places of applying the load in order to prevent the influence of the torque (Gomon et al., 2019). The scheme of the experimental installation for oblique bending tests is shown in Fig. 3.
Fig. 3 Scheme of the experimental installation for testing beams B-3 ... B-6 for oblique bending.

The scheme shown in Fig. 3 allows fully withstand destructive loads for oblique bending at angles of inclination 10° and 25°.

RESULTS AND DISCUSSION

The results of deformation measurements were processed after the experimental tests. An analysis of the destructive loads of the beams was also carried out. The result of such analysis is shown in Table 2. In each series of beam experiments, 5 experiments were conducted (DBN B.2.6-161: 2017). Averaged values were used in subsequent calculations.

<table>
<thead>
<tr>
<th>Sample series</th>
<th>Destructive loads, F, kN</th>
<th>Character of work</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-1</td>
<td>12.2</td>
<td>Direct bend</td>
</tr>
<tr>
<td>B-2</td>
<td>12.3</td>
<td>Direct bend</td>
</tr>
<tr>
<td>B-3</td>
<td>22.0</td>
<td>Oblique bend</td>
</tr>
<tr>
<td>B-4</td>
<td>21.9</td>
<td>Oblique bend</td>
</tr>
<tr>
<td>B-5</td>
<td>15.0</td>
<td>Oblique bend</td>
</tr>
<tr>
<td>B-6</td>
<td>15.1</td>
<td>Oblique bend</td>
</tr>
</tbody>
</table>
The analysis of experimental studies showed that during direct bending the deflections increased in proportion to increase the load. The graph of the beam bending dependence on the load for direct bending is shown in Fig. 4.

![Fig. 4 Experimental full bending of beams B-1 and B-2.](image)

Bending was measured in the horizontal (Y-Y axis) and vertical (Z-Z axis) directions during oblique bending tests:

\[ w = \sqrt{w_Y^2 + w_Z^2} \]  

(1)

Where: \( w_Y \) – bending in the axis direction Y-Y, mm,  
\( w_Z \) – bending in the axis direction Z-Z, mm.

The results of test experiments showed that the bending increased with increasing the load. An increase in inclination angle led to a faster growth of deflections.

The graph of the dependence of total deflections during oblique bending on bending moments is shown in Fig. 5.

![Fig. 5 Experimental full deflections of beams B-3...B-6.](image)

Standard deviation of experimental values of the moment at the ultimate deflection of beams series B-3...B-6 according to results was less than 10% and coefficient of variations approximately 2.6%.

After carrying out experiments and processing data the calculation of the beam characteristics was performed using the Lira SAPR software package. The beam model was constructed using 10x10x10 mm volumetric finite elements. Such samples allowed to obtain more accurate results. Type of finite elements 36. In order to determine the optimal type of
stiffness, it was decided to perform a calculation in the "Lira SAPR" software complex with several of their values:

1) taking into account orthotropy (Stiffness type 1);
2) excluding orthotropy (Stiffness type 2);
3) taking nonlinearity into account (Stiffness type 3).

The following wood characteristics were used for the first type of stiffness according to (DBN B.2.6-161: 2017, EN 408:2010):

\[ E_y = 12000 \text{ MPa; } E_z = 400 \text{ MPa; } \nu_{yz} = 0.018; \nu_{xy} = 0.45; \rho = 460 \text{ kg/m}^3 \]

It is necessary to ensure that the condition is fulfilled for setting the given characteristics:

\[ \frac{E_z}{\nu_{yz}} = \frac{E_y}{\nu_{xy}} \]  

(2)

The following wood characteristics were used for calculations using the second type of stiffness according to (DBN B.2.6-161: 2017, EN 408:2010): \( E_y = 12000 \text{ MPa; } \nu_{zy} = 0.45; \rho = 460 \text{ kg/m}^3 \).

Nonlinearity was considered by constructing a nonlinear deformation diagram of the material (pine wood) (Gomon et al., 2022, Yasniy et al., 2022), which is shown in Fig. 6. At the same time, the following characteristics of the material were used: \( \nu_{zy} = 0.45; \rho = 510 \text{ kg/m}^3 \).

![Deformation diagram of pine wood.](image)

The comparison of the experimental values with the theoretical values was performed at the point of occurrence the maximum allowable full deflection of the test beams. According to current regulatory documents (DBN B.2.6-161: 2017, DSTU B V.1.2-3:2006), the maximum allowable beam deflection is \( l/150 \):

\[ w_{fin} = \frac{l}{150} = \frac{1500}{150} = 10 \text{ mm}, \]  

(3)

Where: \( l \) – beam span.
The deflections ranged from 44 mm under a load of 1 kN to 578 mm under a destructive load for direct bending using the first type of stiffness. All such deflections significantly exceed the experimental values. Therefore, this type was not taken into account in the further analysis.

The values of deflections by use two other types of stiffness are shown in Fig. 7. Also, this graph shows the averaged experimental deflection-bending moment diagram for beams B-1 and B-2.

Fig. 7 Deflections of beams during direct bending.

According to the graph, the closest to the experimental deflection values are values calculated in the "Lira SAPR" software complex using stiffness of type 2 (excluding orthotropy).

The displacement isofields of beam B-1 according to stiffness type 2 and 3 at a bending moment of 2.5 kNm are shown in Fig. 8 and 9, respectively.

Fig. 8 Isofield of beam B-1 movements along Z axis at a moment of 2.5 kNm using the 2nd type of stiffness.
Fig. 9 Isofield of beam B-1 movements along Z axis at a moment of 2.5 kNm using the 3rd type of stiffness.

Full beam deflections of oblique bending (angle of inclination 10°) using stiffness type 1 were 32 mm under a load of 1 kN and 426 mm under a load of 13 kN. These values significantly exceeded the experimental values. Therefore, this type of stiffness was not further analyzed, as well as with direct bending. The displacement of the beam with the other two types of stiffness are shown in the graphs (Fig. 10, Fig. 11) below at different angles of inclination.

Fig. 10 Full deflections of the beams during an oblique bending at the angle of inclination of 10°.
Fig. 11 Full deflections of the beams during an oblique bending at the angle of inclination of 25°.

The displacement isofields of beam B-3 for the second type of stiffness at a moment of 3.25 kNm are shown in Fig. 12, Fig. 13.

A comparison of the averaged experimental diagram of the dependence of the deflections on bending moments with similar ones calculated in the "Lira" software complex is shown in Table 3. The data are given when the maximum allowable beam deflections occur.

The value of the estimated destructive load of beams for the 25-degree angle of inclination was 3.76 kNm, for the 10-degree angle – 5.51 kNm.
The deflection values calculated in the "Lira" software complex using stiffness type 2 are close to corresponding experimental values according to the Table 3. Deviation errors in such cases were less than 16%. Whereas the values of the deflections using the third type of stiffness differ significantly from the experimental values (error up to 54.7%).

EUROCODE 5:2004 and DBN B.2.6-161:2017 proposed a method of calculating wooden beams for oblique bending. This technique does not consider the elastic-plastic properties of wood according to full deformation diagram (Fig. 6). In Eurocode 5:2004 and DBN B.2.6-161:2017 it is presented that wood works only in the elastic stage of work. Our methodology takes into account the elastic-plastic properties of wood.

Kulman et al. (2019) presented simulation of the work of glued wooden beams with and without prestressing by the finite element method. The advantage of this work is that the bearing capacity and deflections of bending elements were determined. The main drawback is the lack of experimental studies. Modeling of beam operation was carried out only under direct bending. In this article, the work of wood was simulated only in the elastic stage. And this does not correspond to its actual work, since wood works as an elastic-plastic material.

Vahedian et al. (2019) conducted experimental studies of reinforced wooden beams and proposed a methodology for their calculation. As advantage of this paper is that the bearing capacity and deflections of flexural elements were experimentally and theoretically determined. The convergence of the results was satisfactory. The study of wooden beams work takes place only under direct bending. It was not held for the oblique.

**CONCLUSION**

New data regarding the load-bearing capacity and deformability of wood in oblique and direct bending conditions were obtained as a result of experimental and theoretical studies. Based on the obtained results, the following conclusions were made:

- the ultimate beam deflections at an inclination angle of 10° occur at an average moment value of 2.05 kNm, at an inclination angle of 25° - 1.6 kNm;
- deflection values calculated by using the second type of stiffness (excluding orthotropy) in the software complex "Lira SAPR" are the closest to the experimental;
- the result of the calculation of the work of beams for direct bending without orthotropy in the "Lira SAPR" software complex showed that the deviation error of deflections of experimental results is 10%, while for the works on oblique bending such error was 6.8% and 15.8% at angles of inclination of 10° and 25°, respectively;
- taking orthotropy into account for the calculation beams for oblique and direct bending in the "Lira SAPR" software complex increases doubles the value of deflections on average;
- nonlinearity significantly increases the value of deflections.
The results of the research can be used in the design of beams made by solid wood of rectangular cross-section under oblique bending. According to results it was established that LIRA software complex can be used to simulate bending deformations of building materials before their use in building construction where can be possible not only direct bending forces but also oblique ones. The best accuracy of modeling and estimation of ultimate beam deflections under the destruction load, which are closest to the real value, is achieved at smaller angles of inclination of oblique bending by using the second type of stiffness (excluding orthotropy). For such bending, the deviation of the experimental data $M_1$ with the calculation results is no more than 16%.

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